



HAL
open science

Heterogeneous Beliefs and Instability Heterogeneous Beliefs and Instability

Laurence Lasselle, Serge Svizzero, Clement Allan Tisdell

► **To cite this version:**

Laurence Lasselle, Serge Svizzero, Clement Allan Tisdell. Heterogeneous Beliefs and Instability Heterogeneous Beliefs and Instability. 2002. hal-02164340

HAL Id: hal-02164340

<https://hal.univ-reunion.fr/hal-02164340v1>

Preprint submitted on 25 Jun 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Heterogeneous Beliefs and Instability

Laurence Lasselle*

Department of Economics, University of St. Andrews, U.K.

Serge Svizzero

CERESUR, University of La Réunion, France

Clem Tisdell

Department of Economics, University of Queensland, Australia

Abstract

While Rational Expectations have dominated the paradigm of expectations formation, they have been more recently challenged on the empirical ground such as, for instance, in the dynamics of the exchange rate. This challenge has led to the introduction of heterogeneous expectations in economic modeling. More specifically, the forecasts of the market participants have been drawn from competing views. Two behaviours are usually considered: agents are either fundamentalist or chartist. Moreover, the possibility of switching from one behaviour to the other one is also assumed.

In a simple cobweb model, we study the dynamics associated with different endogenous switching process based on the path of prices. We provide an example with an asymmetric endogenous switching process built on the dynamics of past prices. This example confirms the widespread belief that fundamentalist market behaviour as compared with that of chartist tends to promote market stability.

J.E.L. numbers: D84, E3

Key words: bounded rationality, chartists, chaos, fundamentalists, rational expectations.

Corresponding author: University of St Andrews, Economics Department, St. Andrews, Fife, KY 169AL, UK.

E-mail: LL5@st-andrews.ac.uk, Tel: 00 44 1334 462 451, Fax: 00 44 1334 462 444.

Heterogeneous Beliefs and Instability

1. Introduction

It is well known that in economics and finance, expectations play a key role in modeling dynamic phenomena. The empirical literature demonstrates that very often the macroeconomics fundamentals are not able to explain the large and persistent movements of some economic variables such as securities prices or exchange rates. A famous example is given by the large appreciation of the USD between 1980 and 1985. There is a wide agreement in explaining that this appreciation came from an increased demand for USD but there is no agreement about why investors suddenly found US assets more attractive than others. In fact, neither models based on economic fundamentals, nor simple time-series models, nor the forecasts of market participants as reflected in the forward discount or in survey data, seem to be able to give good predictions. Even after the fact, the proportion of exchange rate movements that can be explained is very low.

Frankel and Froot (1990a) recalled that three explanations had been given. Firstly, the «safe haven hypothesis» attributed the shift in demand to an increase in the perceived safety of US assets relative to other countries assets. Secondly, according to the speculative bubble hypothesis, there was a self-confirming increase in the expected rate of USD appreciation. Thirdly, the monetarist explanation was based on fundamentals. USD appreciated either because the expected inflation rate differential increased or because the real

interest rate differential increased. Even if the latter explanation appeared to be the best one, it was not very powerful in explaining the path taken by the USD. As a consequence, a number of researchers have deviated from economic models based upon fundamentals, i.e. from the Rational Expectations (RE) paradigm.

Following the seminal papers by Muth (1961) and Lucas (1971), the RE hypothesis has dominated the paradigm in expectations formation. Under this specification, agents' subjective expectations equal the objective mathematical expectations conditional on available information. The RE hypothesis includes in fact two joined assumptions. On the one hand, agents are assumed to have a perfect knowledge of "the economic model", i.e. of the underlying market equilibrium equations. On the other hand, they are assumed to use this model to compute the RE forecast. Both assumptions have been challenged.

Indeed, the literature on bounded rationality has put forward two important criticisms. It is unrealistic to assume that agents know the "economic model"; it seems more reasonable to assume that their expectations are based on time-series observations. It has been shown that, in several economic areas,¹ naive forecasting tends to give more accurate predictions than forecasts based on models even if it cannot as a rule predict turning point. Moreover, even if the agents know the underlying economic model, something else is necessary to obtain the RE. Indeed, the agents also need to have perfect knowledge about

¹ See for instance Witt and Witt, 1995, pp 469-470.

the beliefs of all the other agents in the economy to coordinate their actions on the same RE equilibrium.

Challenging the RE hypothesis leads to two questions.

The first question is how we should model expectations given the fact that it is hard to observe or obtain information about individual expectations in real markets. Of course in theory two polar cases exist: we may assume either full rationality with agents deriving optimal forecast from economic theory or bounded rationality with agents only observing time-series and using simple habitual rule of thumb predictors. Several authors such as Frankel and Froot (1990a, 1990b) answered to this question in creating a new research area where it is assumed that the forecasts of the market participants are drawn from competing views, i.e. heterogeneous expectations are introduced in economic modeling.

The second question is whether heterogeneity in expectations does contribute to excess price volatility as that observed in stock market or exchange rate. In other words, the question is whether agents are able to learn and coordinate on a RE equilibrium in a heterogeneous world.

The aim of this paper is to study the links between heterogeneous beliefs and stability. The paper is organised as follows. Two typical behaviours, namely fundamentalist and chartist, are described in section 2. In section 3 we survey the literature which explains why and how the weight of each behaviour may

change over time. In section 4, using a simple cobweb economy, we obtain different dynamical paths according to assumptions on the forecasting rules and the weighting.

2. Fundamentalists vs. Chartists

Many heterogeneous agents models assume that two kinds of agents exist. The question that arises is on which criteria we could distinguish between both groups of agents.

In fact, besides and even before the RE paradigm, it has long been remarked that if there exists traders who tend to forecast by extrapolating recent trends, i.e. traders who have bandwagon expectations or herd behaviour, then their actions can exacerbate swings in the price. Evidence from survey data shows that at short horizons, respondents tend to forecast by extrapolating recent trends while at long horizons they tend to forecast a return to a long-run equilibrium. One way of distinguishing empirically between the shorter and the longer-term expectations is to examine the weight survey respondents place on past prices in forming their expectations at different time horizons. Frankel and Froot (1990b) considered three standard models of expectations - extrapolative, regressive, and adaptive- and in all three cases, short-term and long-term expectations behave very differently from one another. Therefore, they associated the longer-term expectations, which are consistently

stabilizing, with the *fundamentalists*,² and the short-term forecasts, which seem to have a destabilizing nature, with the *chartists*.³

While it seems to be correct that chartist behaviour is more likely to be destabilizing than fundamentalism, in some cases both behaviours are consistent with stability. This appears for instance in the naive cobweb model when the supply curve shows less responsive than the demand curve to price, chartist, i.e. projectionist behaviour results in market stability.⁴

The behaviour of both groups can be explained in two ways.

First, both groups behave in different manner because they have different information sets. Therefore, each agent is acting rationally subject to certain constraints. The information set of fundamentalists includes fundamentals; thus they base their expectations according to a model that consists of fundamentals. For chartists, their information set contains only the time-series of the expected variable, e.g. the price. So, they use the past history of the price to detect patterns into which they extrapolate the future. However, in this explanation, information is considered as static while it is dynamic, i.e. it may not be considered as given. Indeed, motives influence the type and amount of information gathered. In other words, fundamentalists often collect different types of information to chartists.

² Fundamentalists are also called *arbitrageurs*.

³ Chartists are also called *projectionists*, *technical analysts*, *bandwagon traders* or *noise traders*.

The second way extends the previous analysis and so is better than it. Even when agents have the same information set they may act differently. This may be the case because either they draw a different set of inferences from the same information set or they have different goals, including different attitudes to contending with risk or uncertainty.

Fundamentalists look for fundamental determinants of the price. They calculate an equilibrium price consistent with these fundamentals, and expect that the current price will gradually move towards its equilibrium value:

$$\hat{p}_t^f = p_{t-1} + f(p^* - p_{t-1})$$

where \hat{p}_t^f is the period t price expected by fundamentalists, $f(\bullet)$ is an increasing function and p^* is the equilibrium price.

In the simplest case, they predict that the price will always be equal to the RE equilibrium steady state price, i.e. $\hat{p}_t^f = p^*$.

Chartists base their forecasts on technical analysis, which is mainly a study of past prices to detect patterns that can be projected in the future. They use various extrapolative models that can be summarised under the following general formulation: $\hat{p}_t^c = \phi(p_{t-1}, p_{t-2}, \dots)$

⁴ See Lasselle, Svizzero, and Tisdell, 2001.

where \hat{p}_t^c is the period t price expected by chartists, $\phi(\bullet)$ is an unspecified function.

In the simplest case chartists have naive expectations which represent a rule of thumb forecasting rule, i.e. $\hat{p}_t^c = p_{t-1}$.

3. Switching between fundamentalists and chartists

The change in the price expected by the market can be written as a weighted average of the two groups' expectations: $\hat{p}_t = \alpha_t \hat{p}_t^f + (1 - \alpha_t) \hat{p}_t^c$

where $\alpha_t \in [0,1]$ is the weight associated with fundamentalists and $(1 - \alpha_t)$ the weight of chartists. If α_t is constant over time, this means that the weight of the two groups remains the same and also that switching is impossible, i.e. it is not possible for a fundamentalist to become a chartist and vice versa. However, much evidence supports the possibility of switching.

Indeed, Frankel and Froot (1990a) recalled that theories as well as practice confirm the possibility of switching. On the one hand, the model of speculative bubbles says that over the period 1981-1985, the market shifted weight away from fundamentalists and toward chartists. On the other hand, the *Euromoney Magazine* showed from a survey on techniques used by forecasting services that from 1978 to 1988, the positions between fundamentalists and chartists were reversed. In both cases this shift explained the USD pattern during the period.

Once the weighting is admitted to be able to vary, it remains to explain how it could change. Several strategies have been adopted.

The first strategy consists of stochastic switching.

Vigfusson (1996) noted that this weighting is unobserved, and then the model could not be estimated or tested using standard techniques. To overcome these difficulties and to test the model, he used Markov regime-switching techniques. He defined the two groups' different methods of forecasting as regimes and rewrote the model as a regime-switching model. Kaizoji (1999) also used transition probabilities. Indeed, he considered that it was not possible to get information on the expectations formation and decision-making of all the traders. Therefore and according to the synergetic approach, he treated heterogeneous agents as a statistical ensemble.

The second strategy is based on endogenous switching. Several cases are possible.

The first case provided by Frankel and Froot (1990b) is simple and obvious. Portfolio managers are considered to be the only persons who actually buy and sell on the market. They form their expectations as a weight average of chartists and fundamentalists. Therefore, they update the weights over time according to whether the fundamentalists or the chartists have recently been doing the better forecasting. In other words, the weights that the market gives to the two groups change over time, according to the groups' respective

wealths. The 1981-85 shift was then a natural Bayesian response to the inferior forecasting record of fundamentalists.

This idea may be simply formalised by assuming, as Hommes (1999) did, that prediction rules are updated according to past realised profits. So, there is an evolutionary competition between different forecasting rules and we get:

$$\alpha_t = \exp\left(\beta \pi_{t-1}^f\right) / Z_t$$

where π_{t-1}^f are the net realised profits in period $t-1$ by fundamentalists,

$Z_t = \exp\left(\beta \pi_{t-1}^f\right) + \exp\left(\beta \pi_{t-1}^c\right)$ is a normalization factor so that all fractions

add up to one, β is the intensity of choice, measuring how fast agents switch between different prediction strategies.

If β equals zero, then all fractions are fixed over time. When β tends to infinity, then all agents choose the optimal predictor in each period.

The second case is derived from Frankel and Froot (1990a) and from the empirical observation that the relative weight of the two groups depends on the forecasting horizons. For shorter forecasting horizons, more weight is placed on chartists while the opposite is true for longer forecasting horizons. Bask (1998) considered that, on the one hand, the more uncertain the economy is, the shorter the forecasting horizon is. On the other hand, he pointed out that uncertainty is associated with an inflationary economy. By transitivity, the forecasting horizon depends inversely on the inflation rate.

Then, we get the weight of fundamentalists as a proxy of the forecasting horizon: $\alpha_t = 1/\left[(p_{t-1}/p_{t-2})^2 + \varepsilon\right]$ where ε is positive and small.

When the prices are stable, the forecasting horizon is almost infinite and fundamentalists are dominant; the horizon is myopic if prices are highly unstable.

Even if it is true that agents who remain chartists do work on a shorter horizon, Bask's analysis is however a little bit dubious. Indeed, the usual way to deal with uncertainty is that a higher proportion of individuals become fundamentalists.

The third case comes from De Grauwe *et al.* (1993) and has been adapted in several cases, see Federici and Gandolfo (2000). Although fundamentalists have the same wealth and the same degree of risk aversion, they are assumed to have heterogeneous expectations normally distributed around the equilibrium price p^* . When $p_{t-1} = p^*$, half of the fundamentalists finds that the price is too low and the other half finds it too high, compared to their own estimates. Therefore, the demand of the former is exactly matched by the supply of the latter. In that case, fundamentalists do not influence the price, i.e. $\alpha_t = 0$. When the price deviates from the equilibrium value (in one sense or the other), the two groups of fundamentalists are numerically different so their excess demand is not nil and then they influence the price. Their weight is thus

an increasing function of the deviation of the price from its equilibrium value:

$$\alpha_t = 1 - 1 / \left[1 + \beta (p_{t-1} - p^*)^2 \right].$$

It should be noted that in that case, the weighting changes in a symmetrically manner, which is for sure, a strong assumption.

Most models (based on any switching function) conclude that heterogeneity may cause expectations driven excess price volatility, including chaos. In fact, chaos is possible in a model when the latter has a property of «sensitive dependance on initial conditions»: any two solutions paths with arbitrarily close but not equal starting values diverge at exponential rates. Therefore the future development of the price is essentially unpredictable despite the fact that the underlying model is deterministic and that the solution paths remain within a bounded set.

4. Linear Cobweb dynamics

In this section, we consider the simple cobweb model, i.e. we study the dynamical path of prices on the market for a good. Even if this model is quite simple, it has become a classical example in economic dynamics. Indeed, adaptative as well as rational expectations were first introduced in this model.

In this section, we assume for simplicity that the supply and demand functions are linear (this assumption is removed in the next section).

The demand function is defined by $D(p_t) = a - b p_t$ where a, b are both positive parameters. The supply function is defined by $S(\hat{p}_t) = -c + d \hat{p}_t$ where c, d are both positive parameters. Note that that price expectations denoted by \hat{p}_t are introduced through the producers' behavior.

The stationary equilibrium price is denoted by p^* . It corresponds to a REE if at any period we have $\hat{p}_t = p_t$. In our case, we can easily compute the equilibrium price $p^* = (a + c)/(b + d)$ and the equilibrium quantity $q^* = (ad - bc)/(b + d)$.

Let us define by $\psi(\bullet)$ the expectations function of the producers. This expectations function may depend on the future, current or past prices and it may also include the equilibrium value of the price.

The equilibrium price p^* is said to be *stationary invariant through the expectations* if for any $\psi(\bullet)$ we have $\psi(p^*, p^*, \dots) = p^*$.

We now need to specify the expectations function in order to study the dynamics of the cobweb. It will allow us to establish the link between market stability and the weight of the chartists. Consider a linear expectations function of the equilibrium price and of the price of the previous period, i.e. $\psi(p_{t-1}, p^*) = \alpha p^* + (1 - \alpha)p_{t-1}$.

With our specification, p^* is stationary invariant through the expectation for any value of α . Our dynamical study can be made according to the value taken by α . The two polar cases can first be distinguished and then the intermediary case.

When $\alpha = 0$, all the producers are chartists (like in the traditional cobweb model), i.e. they form their expectations on the previous price. The supply function is upward sloping. The price dynamics is then given by

$$p_t = \frac{a+c}{b} - \frac{d}{b} p_{t-1}. \quad \text{The equilibrium price is still equal to } p^* = (a+c)/(b+d) \text{ and it is unstable if } d > b.$$

When $\alpha = 1$, all the producers are fundamentalists, i.e. they believe that the current price is always equal to its equilibrium value. The supply function has a slope equal to zero; at any price, the producers are willing to sell a constant output equal to $q^* = (ad - bc)/(b + d)$. It is remarkable to note that the stationary equilibrium is stable for any values of the parameters.

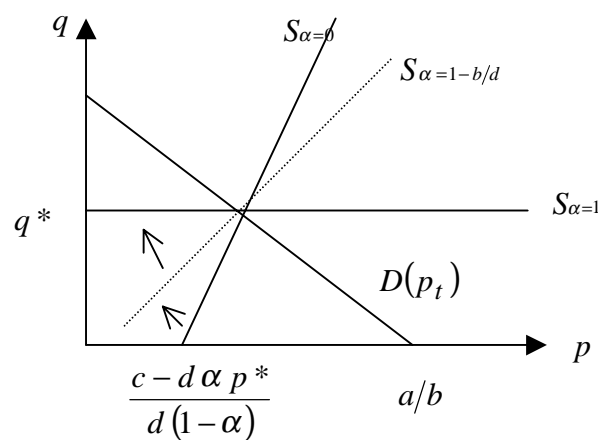
When $0 < \alpha < 1$, both types of producers are on the market. The weight of both groups is linked to the value of α . The higher α is, the bigger the proportion of chartists is. The price dynamics is then defined by:

$$p_t = \frac{(a+c) - \alpha d p^*}{b} - \frac{d}{b} (1 - \alpha) p_{t-1}.$$

The stability of the stationary equilibrium p^* depends on the (constant) slope of the dynamics equal to $-\frac{d}{b}(1 - \alpha)$. Therefore, even when $d > b$, the

equilibrium may be stable⁵ if α is large enough. Indeed, for any α less than $(1 - b/d)$, the equilibrium is unstable; it is stable for any α larger than $(1 - b/d)$. In the special case where $\alpha = 1 - b/d$, a two-period cycle occurs in the neighborhood of the equilibrium price.

We can summarize our results in the following graph.



From this simple model with linear demand and supply functions we can easily deduce that the dynamics is closely related to expectations (as well as the parameters which reflect technology and preferences considered as given). We have shown that the stability conditions of the equilibrium are directly linked to the weight of both groups of producers, namely the weight of chartists relative to the weight of fundamentalists. Indeed, when the number of fundamentalists increases, market stability tends to be promoted.

It remains to study whether this result still holds for a non linear supply function which would thereafter allow us to make α endogenous.

⁵ As we have previously shown, the equilibrium is unstable when all producers are chartists and that this condition is met.

5. Non linear Cobweb dynamics

Our framework is based on the basic cobweb model derived from Hommes (1999). While the demand function is still linear the supply function is not.

The linear demand function is defined by $D(p_t) = A - B p_t$ where A, B are both positive parameters. The supply curve is S-shaped and equals $S(\hat{p}_t) = (\hat{p}_t - 1)^{1/d} + 1$, where d is an odd integer. Such a supply curve can be easily derived from an increasing and convex cost function given by at least a third order polynomial.

When the expectations are not specified, the dynamics is given by:

$$p_t = \frac{A-1}{B} - \frac{1}{B} \left[(\hat{p}_t - 1)^{1/d} \right] \quad (1)$$

When $\hat{p}_t = p_t$, there is perfect foresight and we can then compute the rational expectations steady state denoted by p^* . A convenient feature of the cobweb model is the uniqueness of the RE equilibrium. In the sequel we simulate the cobweb model for different specification of the expectations functions.

First let us assume that expectations are homogeneous. More specifically let us say these expectations are naive or myopic, i.e. when $\hat{p}_t = p_{t-1}$, (1) becomes:

$$p_t = \frac{A-1}{B} - \frac{1}{B} \left[(p_{t-1} - 1)^{1/d} \right]$$

In this case, there is a single group of agents and therefore no switching function. Depending on the values of the parameters (A,B,d) , the REE is stable (see Figure 1 in the Appendix) or unstable.

Second let us assume that expectations are heterogeneous. In this case, two groups of agents coexist: the chartists and the fundamentalists. Each group has his own way to formulate expectations:⁶

- Chartists have myopic expectations: $\hat{p}_t = p_{t-1}$,
- Fundamentalists consider that the expected price is always equal to the steady state price: $\hat{p}_t = p^*$.

Following Hommes (1999) we assume that the weighting of both groups is exogenous. We denote by $\alpha \in [0,1]$ the weight associated with fundamentalists.

(1) becomes:

$$p_t = \frac{A-1}{B} - \frac{1}{B} \left[\left((1-\alpha)p_{t-1} + \alpha p^* - 1 \right)^{1/d} \right] \quad (2)$$

We simulate (2) for a broad range values of parameters A , B and d and also for different weighting, i.e. for various values of α .

In any case, the REE is unique. Depending on the values of the parameters (A,B,d,α) , this REE can be either stable or unstable.

⁶ Many expectations functions are consistent with both behaviours, we consider the simplest functions.

Consider first the case of a stable REE. If there are initially few fundamentalists, it remains stable⁷ when the weight of fundamentalists increases. However, when the REE is initially unstable⁸ for a low α , as soon as the weight of fundamentalists increases, a two-period cycle emerges. Thereafter the REE becomes stable for higher values of α .

The simulation results show that fundamentalists play a role in stabilizing the economy. So, it remains to be shown if the same stabilizing effect is associated with the increasing weight of fundamentalists for any endogenous switching function.

Following Federici and Gandolfo (2000), we now assume that the switching is endogenous and symmetric. (1) is becoming:

$$p_t = \frac{A-1}{B} - \frac{1}{B} \left[((1-\alpha_t)p_{t-1} + \alpha_t p^* - 1)^{1/d} \right] \quad (3)$$

and the switching process, based on the deviation of the price from its equilibrium value, is defined by:

$$\alpha_t = 1 - \frac{1}{1 + \beta(p_{t-1} - p^*)^2}.$$

As soon as the current price is far from the REE price, the weight of chartists decreases. It is important to note that this process is symmetric, i.e. the

⁷ See Figure 2 in the Appendix where the dynamical path of prices is drawn for a given value of α .

⁸ The instability is provided by the values of the other parameters, namely (A, B, d) .

elasticity of the weighting w.r.t. the price deviation remains the same whatever the sign of the deviation.

From the simulations results we deduce the following (given values of the parameters⁹). If the REE is stable¹⁰ (respectively unstable) for a given initial value of alpha, then it remains stable (respectively unstable) for any α . In other words, the switching does not influence the stability conditions. More specifically, the weight of fundamentalists does not imply the stabilizing effect that appears in the exogenous switching case.

This apparently strange result may be explained by the features of the switching function. Indeed, it appears that, when evaluated at the equilibrium price p^* , this function is equal to zero and therefore it may not have an influence on the slope of the dynamics.

Given the previous result, it appears necessary to keep an endogenous switching process but to remove its symmetry.

Recall that at any period, we must have $\alpha_t \in [0,1]$ and the dynamics of prices is still given by (3).

Our switching process, based on the fluctuations of past prices, is now asymmetric. Its specification depends on a given evolution of past prices. When past prices increased, few number of fundamentalists may turn to be

⁹ However, the results hold for any β .

¹⁰ See Figure 3 in the Appendix.

chartist. When there is a downward trend in prices, a large number of chartists may turn to be fundamentalist.

In our simulations, the following asymmetric specification of α_t has been chosen:

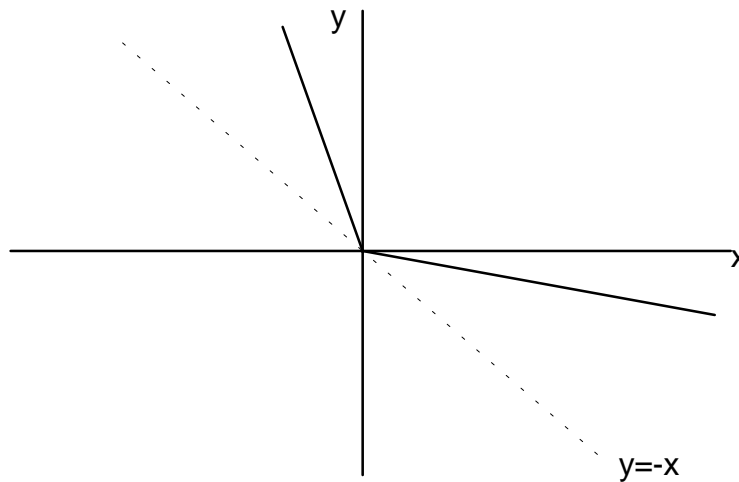
When the past prices increased ($p_{t-1} - p_{t-2} > 0$), there are more chartists, i.e. α_t decreases, but we assume that α_t decreases slowly.

When the past prices decreased ($p_{t-1} - p_{t-2} < 0$), there are less chartists, i.e. α_t increases, but we assume that α_t increases faster.

Consequently, the dynamics of the weighting is clearly asymmetric.

For ease of understanding, let us introduce the following notations:

$x = p_{t-1} - p_{t-2}$ and $y = \alpha_t - \alpha_{t-1}$ and draw a diagram of our specification.



Each branch of the lines represents the switching process and is given by:

$$y = -x(1 + \varepsilon)$$

with $\varepsilon = -\gamma$ if $x > 0$,

and with $\varepsilon = 1/\gamma$ if $x < 0$,

and in both cases $\gamma \in [0,1]$.

The dynamics now includes two dynamical variables (the weighting and the price). We deal with a two-dimensional dynamical system described by:

- the dynamics on prices (3)
- the dynamics on α_t (i.e. the switching process).

The results from simulations are the following.¹¹ For any values of the parameters, the unique REE is stable. On the transition path, the values taken by α_t are increasing, i.e. the stability is associated with an increase of the weight of fundamentalists.

6. Concluding comments

In a simple cobweb model we have shown that the introduction of heterogeneous beliefs and of the possibility of switching of behaviour allow the economy to shift from instability to stability. This shift is attributed to the increase of the weight of fundamentalists in the case of an exogenous switching. While we have focused our attention on how the weight of each group influences the stability/instability result, it should be noted that it will also have an impact on the speed of convergence to the REE.

¹¹ See Figure 4 in the Appendix.

When we have considered an endogenous and symmetric switching process, the economy may remain unstable.

However, when the endogenous switching process is asymmetric and based on past prices, then stability is always obtained and is associated with an increase of the number of fundamentalists. This latter result confirms the widespread belief that fundamentalist market behaviour as compared with that of chartist tends to promote market stability.

Acknowledgements

This paper was presented at the 2001 MMF (Money, Macro and Finance Group) conference in Belfast, UK. We wish to thank the participants of the MMF conference for their constructive comments. Errors remain ours.

References

Bask, M. (1998), Chartists and fundamentalists in the foreign exchange market: forecasting horizons and exchange rate dynamics, Working Paper 465d, Department of Economics, Umea University.

De Grauwe, P., Dewachter, H. and Embrechts, M. (1993), *Exchange rate theory: chaotic models of exchange rate*, Oxford, Blackwell.

Federici, D. and Gandolfo, G. (2000), Is there chaos in the Italian exchange rate? mimeo.

Frankel, J.A. and Froot, K. A. (1990a), Chartists, fundamentalists and the demand for dollars, in *Private behaviour and government policy in interdependent economies*, edited by A. S. Courakis and M. P. Taylor, Clarendon Press: Oxford.

Frankel, J.A. and Froot, K. A. (1990b), Chartists, fundamentalists, and trading in the foreign exchange market, *American Economic Review* **80**(2), 181-185.

Hommes, C. (1999), Cobweb dynamics under bounded rationality. CeNDEF Working Paper 99-05, University of Amsterdam.

Kaizoji, T. (1999), Complex dynamics of speculative price, *Complexity International* **6**, 21 pages.

Lasselle, L., Svizzero, S., and Tisdell, C. (2001), Diversity, globalisation and market stability, forthcoming in *Economia Internazionale*.

Lucas, R. E. (1971), Econometric testing of the natural rate hypothesis, in *The econometrics of price determination conference*, Board of governors of the federal reserve system and social science research council, edited by O. Eckstein.

Muth, J. F. (1961), Rational expectations and the theory of price movements, *Econometrica* **29**, 315-335.

Vigfusson, R. (1996), Switching between chartists and fundamentalists: a Markov switching approach, Bank of Canada, Working Paper 96-1.

Witt, C. A. and Witt, S. F. (1995), Forecasting tourism demand: a review of empirical research, *International Journal of Forecasting* **11**(3), 447-475.

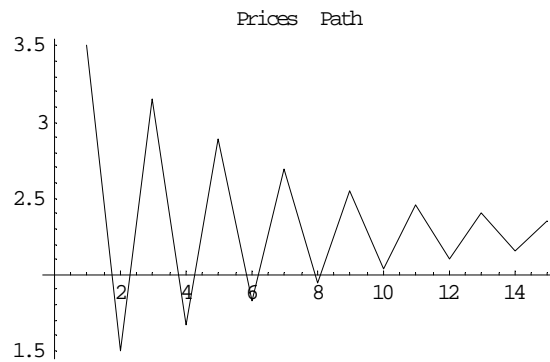
Appendix

In the following simulations, the values of the parameters are:

$A = 2.5$, $B = 0.2$, and $d = 5$. Please note that the initial price is denoted by p_1 where 1 is the initial period of time.

Figure 1: homogeneous expectations

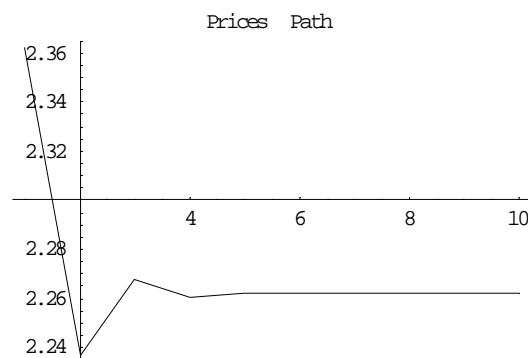
$$p_1 = 3.5$$



Value of the REE, $p^* = 2.26189$.

Figure 2: heterogeneous expectations and exogenous switching process

Let $\alpha = 0.7$ and $p_1 = 2.36189$



Value of the REE, $p^* = 2.26189$.

Figure 3: heterogeneous expectations and endogenous symmetric switching process

Let $\beta = 0.3$ and $p_1 = 2.36189$



Value of the REE, $p^* = 2.26189$

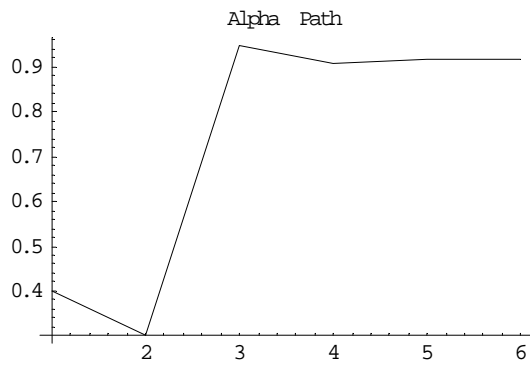
Figure 4: heterogeneous expectations and endogenous asymmetric switching process

$\gamma = 0.5$, $p_1 = 2.2$, $p_2 = 2.4$, $\alpha_2 = 0.1$

$p^* = 2.26189$



Prices Path: {2.2, 2.4, 2.18398, 2.26526, 2.26163, 2.26191, 2.26189}



Values of the weighting taken on the path: {0.4, 0.3, 0.948047, 0.907411, 0.91828, 0.918142, 0.918199, 0.918198, 0.918198}

