A mathematical model of Collective Intelligence

Federica Stefanelli, Enrico Imbimbo, Timoteo Carletti, Alessio Guarino, Francois Burette, Andrea Guazzini

A mathematical model of Collective Intelligence

FEDERICA STEFANELLI, ENRICO IMBIMBO, University of Florence, Italy
TIMOTEO CARLETTI, University of Namur, Belgium
ALESSIO GUARINO, University of Reunion, France
FRANCOIS BURETTE, Lyce Roland Garros - Acadmie de La Runion, France
ANDREA GUAZZINI, University of Florence, Italy

1. INTRODUCTION

Human beings, as social animals, can organize themselves into groups to solve problems that a single individual is not able to solve by working alone [Smith 1994]. Indeed, merging their problem representations [Heylighen 1999], the group members' become capable of overcoming the hurdle that each one member does not exceed alone. The social problem-solving that the group implements in order to solve higher-level tasks is the result of the group members' cooperation and of their abilities in sharing knowledge. Thus, people tend to turn to groups when they have to solve complex problems because they believe that groups have better decision-making skills than a single individual [Forsyth 2018]. The measure of the difference between the rate of success of the group in a specific problem and the average of the rate of the success of the group's agents in the same issue can be considered the *collective intelligence*. The *collective intelligence* is an emergent property of groups, not reducible to the simple sum of its members' individual intelligence. The general factor able to explain the "groups performance on a wide variety of tasks" [Woolley et al. 2010] is the result of the complex interaction among many factors, such as the member characteristics, the group structure that regulate collective behavior [Woolley et al. 2015], the context in which the group work [Barlow and Dennis 2016], the cognitive processes underlying the social problem-solving reasoning [Heylighen 1999], the average of members' individual intelligence [Bates and Gupta 2017] and the structure [Credé and Howardson 2017; Lam 1997], and the complexity of the problem that it has to solve [Capraro and Cococcioni 2016; Guazzini et al. 2015; Moore and Tenbrunsel 2014]. Although the literature identifies a relationship between *collective intelligence* and task complexity, the limited number of studies in this field make the results still elusive. In particular, it is possible to postulate the existence of a complex and nonlinear interaction between the potential of the group (average of members' intelligence) and the complexity of the problem that the group has to solve that may explain the variance of group performance. Thus, it could be of interest in the field of the study of *collective intelligence* to clarify the relationship between the two variables named above in order to propose a model useful to explain the group performance. To better understand how the relationship between group members' intelligence and task complexity influences the group performance, it would be necessary to analyze the cognitive processes underlying the social problem-solving reasoning that the group implements. In this regard, Heylighen [1999], thorough an interesting formal model, suggests that groups, solving a task, develop a Collective Mental Map (CMM) as a product of the interaction between some psychosocial processes, such as the cross-cueing [Meudell et al. 1995] and the information and knowledge sharing. The Heylighen's framework allows studying the *collective intelligence* dynamics taking into account the merge of the group members' representations of the problem in a single representation (MM). The problems' mental maps are composed of a set of problem states, a set of possible steps for the solution of the task, and a preference fitness criterion for selecting the preferred actions. Here, adopting the Heylighen's framework, we propose a mathematical model of *collective intelligence* useful to shed in light how the interaction between the average of group members' intelligence and the complexity of the task can influence the group performance.

2. MODEL

In order to study the complex relationship among task difficulty, group members' intelligence, and *collective intelligence*, we designed a computational model based on the Heylighen theoretical framework. Adopting the Heylighen topological metaphor of human knowledge, the i-agent has been represented by a vector made of D entries (knowledge nodes), $\vec{K}^{(i)} = (K_1^{(i)}, \dots, K_D^{(i)})$. Each element of the knowledge vector could assume values in the range $[0, 1]$; the smaller (resp. the larger) was the value, the lower (resp. the higher) was the knowledge on this specific topic. As a consequence, the total knowledge of an agent can be obtained by the sum of the elements of its knowledge vector, $IQ^{(i)} = \sum_{j=1}^{D} K_j^{(i)}$; thus the latter can be used as a proxy of the Intelligence Quotient of the agent. For the sake of simplicity, we assumed the agent knowledge as a random variable uniformly distributed in $[0, 1]$. We adopted the same rationale in order to represent the tasks that an agent, or a group, were to solve. We assumed a task made by a D dimensional vector, representing the D topics one has to master to achieve the task. In the present work, for the sake of simplicity, we studied the simplified case in which the components of a task have all the same values τ, so having $\vec{\tau} = (\tau, \dots, \tau) \in [0, 1]^D$. Finally, an agent was able to solve a task with difficulty τ, if all the entries in its knowledge vector were larger than τ, namely $\min_j K_j^{(i)} \geq \tau$. Let us observe that the dimension D indirectly participates to make a task hard or not; indeed, if D is large, it can be difficult (i.e. less probable) for the agents to have all the entries of their knowledge vector larger than τ. The last required ingredient is a set of rules driving the merge of different mental maps (agents' knowledge) into a common one (group knowledge) in order to model the group task solving process. One of the simplest way to implement the Heylighen framework in the absence of any communication issue and/or social hierarchy (e.g., status, roles), is to assume that agents "juxtapose" their knowledge, that is the Collective Mental Map will result to be a D-dimensional vector, $\vec{G} = (G_1, \dots, G_D)$, whose entries are the "best ones" among the agents, more precisely $G_j = \max_i K_j^{(i)}$. Based on the above, a group was able to solve a task of difficulty $\tau \in [0, 1]$ if $\min_j G_j \geq \tau$. Clearly if the group contained agents capable to solve by their own a task of a given difficulty, the group would also do the same, but in this case, the *collective intelligence* would be null because there was not an added value to be together. On the other hand, a group made by agents unable to solve alone a task of a given difficulty, but excelling in sufficiently many different topics, could perform well and solve a problem where each agent would have been failed. In this latter case, one can consider such achievement an emergent property of the group and assign a large *collective intelligence*. Given a task of difficulty $\tau \in [0, 1]$ in a knowledge space of D dimensions, we can define the *collective intelligence* $CI(\tau, D)$ of a group as the difference between the rate of success of the group R, and the rate of success of the average agent composing the group $R_{\langle A \rangle}$ (Eq. 1).

$$CI(\tau, D) = R(\tau, D) - R_{\langle A \rangle}(\tau, D) \tag{1}$$

In this way this function depends on τ and D, it is non-negative and positive values are associated with tasks too hard for the individual agent while solvable by the group.

2.1 Numerical Simulation

In each step of the numerical simulations, a group of size N composed by randomly generated agents faced with tasks of increasing complexity τ, both as a group (considering the collective mental map resulting from the merging of the agents' knowledge vectors), as well as individually (i.e., each agent was let to try to solve the task alone). To reduce the stochastic effects of the model each point has been obtained as the average over 1000 independent replicas. Finally the average value of *collective intelligence*, as computed by the equation 1, was recorded.

3. RESULTS

The main aim of the present work was to investigate the complex relationships among task difficulty, group members' intelligence, and *collective intelligence*, however using a uniform distribution to generate agents knowledge would induce a constant average IQ. So to overcome this limitation we decided to adopt a tent distribution for the agent knowledge; let us recall that the tent distribution depends on a parameter, tuning which the average agent IQ will vary. The figure 1 confirms a non-linear relation between average agents IQ and *collective intelligence*. Three different regimes depending on the task difficulty can be appreciated: when the task is easy, the *collective intelligence* rapidly decreases with the increases in the average agents IQ. The opposite happens when the task difficulty is high, while for tasks of intermediate value a maximum in CI emerges for an optimal value of group IQ.

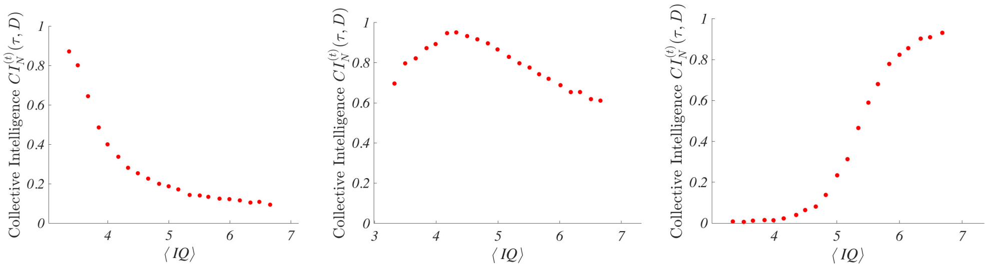

Fig. 1. The *collective intelligence* as a function of the average agent IQ in a small group. We show the CI for a group made by $N = 10$ agents solving an easy task (left panel $\tau = 0.1$), a simple level task (middle panel $\tau = 0.3$) or a more difficult one (right panel $\tau = 0.6$), in all cases $D = 10$.

Finally, generating the agents using a tent distribution, we studied *collective intelligence* as a function of τ and D. Results presented in Fig. 2 suggest that for small τ, namely once the task are simple, both the group and the average agent perform well, and thus the *collective intelligence* appears to be small. Within this regime, the added value of the group is negligible concerning the average agent. Once τ increases, the agent starts to do poorly while the group keeps is a high level of success, determining a large value for the *collective intelligence*. Once τ gets even larger also the groups' rate deteriorates and the *collective intelligence* drop again to 0. Increasing the dimension space D makes the emergence of the *collective intelligence* even sharper. Thus, considering a system where every agent i is characterized by the same IQ_i, is possible to highlights three different regimes emerging from the interaction between the task dimensionality D, and the task complexity τ.

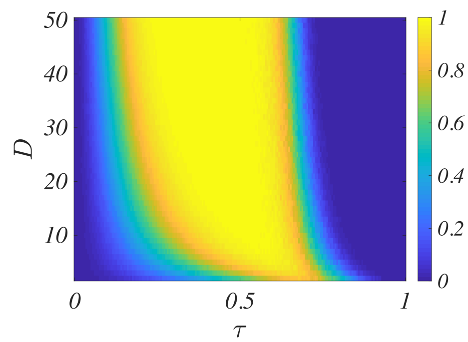

Fig. 2. The *collective intelligence* as a function of the task difficulty, τ and D, for a group made of $N = 10$ agents.

REFERENCES

Jordan B Barlow and Alan R Dennis. 2016. Not as smart as we think: A study of collective intelligence in virtual groups. *Journal of Management Information Systems* 33, 3 (2016), 684–712.

Timothy C Bates and Shivani Gupta. 2017. Smart groups of smart people: Evidence for IQ as the origin of collective intelligence in the performance of human groups. *Intelligence* 60 (2017), 46–56.

Valerio Capraro and Giorgia Cococcioni. 2016. Rethinking spontaneous giving: Extreme time pressure and ego-depletion favor self-regarding reactions. *Scientific reports* 6 (2016), 27219.

Marcus Credé and Garett Howardson. 2017. The structure of group task performance?A second look at ?collective intelligence?: Comment on Woolley et al.(2010). (2017).

Donelson R Forsyth. 2018. *Group dynamics*. Cengage Learning.

Andrea Guazzini, Daniele Vilone, Camillo Donati, Annalisa Nardi, and Zoran Levnajić. 2015. Modeling crowdsourcing as collective problem solving. *Scientific reports* 5 (2015), 16557.

Francis Heylighen. 1999. Collective Intelligence and its Implementation on the Web: algorithms to develop a collective mental map. *Computational & Mathematical Organization Theory* 5, 3 (1999), 253–280.

Simon SK Lam. 1997. The effects of group decision support systems and task structures on group communication and decision quality. *Journal of Management Information Systems* 13, 4 (1997), 193–215.

Peter R Meudell, Graham J Hitch, and MM Boyle. 1995. Collaboration in recall: Do pairs of people cross-cue each other to produce new memories? *The Quarterly Journal of Experimental Psychology* 48, 1 (1995), 141–152.

Celia Moore and Ann E Tenbrunsel. 2014. ?Just think about it?? Cognitive complexity and moral choice. *Organizational Behavior and Human Decision Processes* 123, 2 (2014), 138–149.

John B Smith. 1994. *Collective intelligence in computer-based collaboration*. CRC Press.

Anita Williams Woolley, Ishani Aggarwal, and Thomas W Malone. 2015. Collective intelligence and group performance. *Current Directions in Psychological Science* 24, 6 (2015), 420–424.

Anita Williams Woolley, Christopher F Chabris, Alex Pentland, Nada Hashmi, and Thomas W Malone. 2010. Evidence for a collective intelligence factor in the performance of human groups. *science* 330, 6004 (2010), 686–688.