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Short Communication

A novel non-linear model-based control strategy to improve PEMFC water management – The flatness-based approach

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A B S T R A C T

In the area of PEMFC, water management and thus membrane humidity still remains one of the most challenging issues affecting PEMFC efficiency and lifetime. In this investigation, an innovative method to improve PEMFC water management is presented and a non-linear model-based control strategy is proposed. The novelty of this approach relies upon a simplified PEMFC model combining the benefits of the Differential Flatness Theory. Efficiency and relevance of the proposed water management strategy is confirmed in simulation environment through several controlled scenarios. It was found that in each case, the flatness-based controller successfully regulates the membrane humidity, while avoiding flooding or even membrane drying that can lead to irreversible damage. Furthermore, the novel model demonstrates excellent performance in terms of set-point tracking, disturbances rejection and robustness against parameters uncertainties and measurement noise. Overall, this novel approach appears to be a possible and promising towards improving PEMFC water management issues.

Introduction

Proton Exchange Membrane Fuel Cell (PEMFC) is proven to be a promising environmental friendly technology for portable, mobile and stationary applications. However, several issues still need to be addressed and solved to reduce its cost, extend

its lifetime and improve its efficiency. Among them, water management still remains one of the most challenging problems. Proton Exchange Membrane (PEM) humidity strongly affects PEMFC performance and lifetime. On the one hand, excess of water can cause flooding, which lead to a cell voltage drop. On the other hand, water leaks may dry the membrane and causes irreversible damages. Therefore, to ensure optimal

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Nomenclature			
a	water activities	ca	cathode
F	Faraday constant, C	gen	generated
I	current, A	in	inlet
ℓ	orifice flow constant, $\text{kg s}^{-1} \text{Pa}^{-1}$	m	average
M	molecular mass, kg mol^{-1}	mbr	membrane
n	number of cells	N_2	nitrogen
P	pressure, Pa	O_2	oxygen
R	gas constant, $\text{J kg}^{-1} \text{K}^{-1}$	out	outlet
T	temperature, K	rm	return manifold
V	volume, m^3	sat	saturation
W	mass flow rate, kg s^{-1}	st	stack
		v	vapor
<i>Greek letters</i>		<i>Controller parameters</i>	
λ	membrane water content	k_{ij}	controller tuning parameters
ϕ	relative humidity, %	u	input vector
ω	humidity ratio	x	state vector
		y	output vector
<i>Subscripts</i>		y_{ref}	set-point output vector
a	air	$\psi(\cdot), \varphi_1(\cdot), \varphi_2(\cdot)$	mapping functions

efficiency while avoiding flooding or membrane drying, the PEM has to be properly hydrated.

Many studies have been conducted to elucidate water transport phenomena in PEMFCs. In addition, to improve the understanding of these phenomena, while avoiding additional costs related to experimental studies, numerous models have been developed. A fairly recent and very detailed review on experimental studies and modeling works related to water transport in PEMFC can be consulted in Ref. [1].

However, whereas numerous studies have focused on water transport modeling in PEMFC, a very few studies have been reported in the control of the membrane humidity. Among them, Haddad et al. 2008 [2] proposed an open-loop control of the membrane humidity using the water mole fractions in the inlet gases as manipulated variables. Simulation results showed that an appropriate control of the membrane water content could minimize the electrical energy loss. Zhang et al. 2008 [3] designed a model-based predictive controller to maintain an appropriate water concentration in the cathode, as long as the manipulated variable is the mass flow rate of injected water from humidifier. The proposed controller, tested in simulated environment, allowed the reduction of water concentration fluctuation in the cathode. Hussaini and Wang 2010 [4] proposed a water management strategy based on an intermittent external humidification protocol. This approach, dedicated to low temperature fuel cells operating at constant current densities, aimed to maintain the membrane close to full humidification, while avoiding flow field plate channel flooding. Experimental results showed that appropriate control of dry and humid flows durations improved the fuel cell performance. Sedighzade and Fathian 2011 [5] proposed to manipulate the anode and cathode water mole fractions in views of maintaining a constant voltage and keeping the membrane humidity within an appropriate range, regardless of the cell current. In this investigation, the authors designed a recurrent neural network controller. Simulation results, performed for a single cell PEMFC, showed that the controller was able to track

the voltage set-point, while keeping the membrane water content within its standard limits. Khoeihi and Zarabadipour 2012 [6] proposed an optimal PID controller based on genetic algorithm to improve water management in PEMFC. Herein, the anode and cathode water mole fractions were manipulated to regulate the output cell voltage and maintain the cell water content within an acceptable interval. The controller, implemented in simulation environment, exhibited satisfactory cell voltage tracking capability, while keeping the water content within an appropriate range.

In this paper, a flatness-based approach model is proposed in views of tackling PEMFC water management issues. To ensure optimal PEMFC efficiency, while avoiding flooding or membrane drying, a flatness-based controller is designed to regulate the membrane humidity. By using the Differential Flatness Theory, the control law is explicitly expressed in terms of the so-called flat outputs and a finite number of their derivatives, without integrating any differential equations. In this manner, the controller takes into account the non-linear process behaviors, while avoiding hefty computational exercise. This paper is organized as follows: [PEMFC water management design](#) is dedicated to the flatness-based controller design; in [Simulation-based results](#), the performance of the proposed controller in terms of tracking capability, disturbances rejection ability, robustness against parameter uncertainties and measurement noise, is investigated in simulation environment.

PEMFC water management design

Differential Flatness Theory has been proven to be a very powerful concept, which has been successfully applied to a variety of non-linear systems across various Engineering disciplines ranging from chemical reactor to space robotic [7–16]. Recently, a few works emphasizing benefits of flatness-based controller regarding PEMFC control have been reported [12,17,18].

Differential Flatness Theory principle

Given a non-linear system such as:

$$\dot{x} = f(x, u); x \in \mathbb{R}^n; u \in \mathbb{R}^m \quad (1)$$

where $x = [x_1, x_2, \dots, x_n]^T$ and $u = [u_1, u_2, \dots, u_m]^T$ denote state and input vectors, respectively.

This system is differentially flat, or flat, if and only if, there exists an output vector $y \in \mathbb{R}^m$ called flat output, such as $y = [y_1, y_2, \dots, y_m]^T = \psi(x, \dot{x}, \dots, u^{(\beta)})$ implies [19–21]:

$$\begin{cases} x = \varphi_1(y, \dot{y}, \dots, y^{(\alpha)}) \\ u = \varphi_2(y, \dot{y}, \dots, y^{(\alpha+1)}) \end{cases} \quad (2)$$

α and β are finite numbers of derivatives, $\psi : \mathbb{R}^n \times (\mathbb{R}^m)^{(\beta+1)}$, $\varphi_1 : (\mathbb{R}^m)^\alpha \times \mathbb{R}^n$ and $\varphi_2 : (\mathbb{R}^m)^{(\alpha+1)} \times \mathbb{R}^m$.

Let us recall that a flat system is equivalent to a linear one via an endogenous feedback [19–21]. Therefore, if the above non-linear system is flat, one can find an endogenous feedback such as: $y_i^{(\alpha+1)} = v_i$. In this context, the tracking control problem can be written as:

$$v_i = y_{ref,i}^{(\alpha+1)} + \sum_{j=0}^{\alpha} k_{ij} e_i^{(j)} \quad i = 1, \dots, m \quad (3)$$

$y_{ref,i}$ denotes the desired set-point value for the i th output, $e_i^{(j)} = y_{ref,i}^{(j)} - y_i^{(j)}$ is the tracking error, and parameters k_{ij} are chosen such that the m polynomials $[s^{(\alpha+1)} + \sum_{j=0}^{\alpha} k_{ij} s^j]$ are strictly Hurwitz.

Note that this definition allows expressing all systems' variables and control actions, in terms of flat outputs and a finite number of their derivatives. Flatness-based controllers offer a great trade-off between not optimal linear controllers, such as proportional integral derivative controllers, and high computational cost non-linear controllers such as non-linear model-based predictive control strategies. Indeed, flatness-based controllers take account of the non-linear behavior of the process while avoiding heavy computations.

Flatness-based controller design

In this work, a flatness-based controller was designed to control the membrane humidity based upon the following simplified model:

$$\frac{dm_{w,ca}}{dt} = W_{v,ca,in} - W_{v,ca,out} + W_{v,ca,gen} + W_{v,mbr} \quad (4)$$

where $W_{v,mbr}$ denotes the mass flow rate of water across the membrane, and the inlet mass flow rate of vapor $W_{v,ca,in} = W_{ca,in}(1 - 1/(1 + \omega_{ca,in}))$ is written as a function of the inlet mass flow rate $W_{ca,in}$ and the humidity ratio $\omega_{ca,in}$:

$$\omega_{ca,in} = \frac{M_v \phi_{ca,in} P_{sat}^{T_{ca,in}}}{(P_{ca,in} - \phi_{ca,in} P_{sat}^{T_{ca,in}}) M_a} \quad (5)$$

M_v is the vapor molar mass, the air molar mass $M_a = \mathcal{Y}_{O_2} M_{O_2} + (1 - \mathcal{Y}_{O_2}) M_{N_2}$ depends on the fraction of oxygen in the dry air \mathcal{Y}_{O_2} , the molar mass of oxygen M_{O_2} , and nitrogen, M_{N_2} . The vapor saturation pressure $P_{sat}^{T_{ca,in}}$ is obtained from the equation given in Ref. [22]. $\phi_{ca,in}$ and $P_{ca,in}$ are the relative humidity and pressure of the inlet gas at the cathode, respectively.

The outlet mass flow rate of vapor $W_{v,ca,out} = W_{ca,out}(1 - 1/(1 + \omega_{ca,out}))$ depends on the outlet mass flow rate $W_{ca,out}$ and the humidity ratio $\omega_{ca,out}$:

$$W_{ca,out} = \mathcal{K}_{ca}(P_{ca} - P_{rm}) \quad (6)$$

$$\omega_{ca,out} = \frac{M_v P_{v,ca}}{(P_{O_2} + P_{N_2}) M_a} \quad (7)$$

\mathcal{K}_{ca} is the orifice flow constant, $P_{ca} = P_{O_2} + P_{N_2} + P_{v,ca}$ the cathode total pressure and P_{rm} the return manifold pressure. The partial pressure of oxygen, nitrogen and vapor are obtained according to the ideal gas law such as $P_{O_2} = (m_{O_2} R_{O_2} T_{st})/V_{ca}$, $P_{N_2} = (m_{N_2} R_{N_2} T_{st})/V_{ca}$ and $P_{v,ca} = (m_{v,ca} R_v T_{st})/V_{ca}$. The mass flow rate of generated water $W_{v,ca,gen} = M_v n I_{st}/2F$ is a function of the stack current I_{st} , the number of cells n , and the Faraday constant F .

Considering $x = m_{w,ca}$ as state variable, $u = W_{ca,in}$ as manipulated variable and $y = m_{w,ca}$ as output, it is obvious that $x = \varphi_1(y)$. Besides, from Eq. (4) the control variable can be expressed in terms of the output y and its first time derivative:

$$u = \frac{\left[\dot{y} + \mathcal{K}_{ca} \left(P_{O_2} + P_{N_2} + \frac{y R_v T_{st}}{V_{ca}} - P_{rm} \right) \left(1 - \frac{(P_{O_2} + P_{N_2}) M_a V_{ca}}{y M_v R_v T_{st}} \right) \right]}{(1 - 1/(1 + \omega_{ca,in}))} + \frac{-W_{v,ca,gen} - W_{v,mbr}}{(1 - 1/(1 + \omega_{ca,in}))} = \varphi_2(y, \dot{y}) \quad (8)$$

Since $x = \varphi_1(y)$ and $u = \varphi_2(y, \dot{y})$ and according to Eq. (2), the system Eq. (4) is differentially flat and has y as flat output. Once the system has been proven to be flat, the tracking control problem becomes straightforward, and the closed-loop control law, deduced from Eq. (3) and Eq. (4), is given by:

$$u = \text{Sat}_{u_{min}}^{u_{max}} \left(\frac{y_{ref}^{(1)} + k_1(y_{ref} - y) + W_{v,ca,out} - W_{v,ca,gen} - W_{v,mbr}}{(1 - 1/(1 + \omega_{ca,in}))} \right) \quad (9)$$

where $y_{ref} = m_{ref,w,ca}$ denotes the desired mass of water, and $\text{Sat} : \mathbb{R} \rightarrow \mathbb{R}$ denotes a saturation map, such as:

$$\text{Sat}_{u_{min}}^{u_{max}}(u) = \begin{cases} u_{min} & \text{if } u \leq u_{min} \\ u_{max} & \text{if } u \geq u_{max} \\ u & \text{otherwise} \end{cases} \quad (10)$$

Note that the saturation constraints on the manipulated variable are used to ensure safety operation of the PEMFC system.

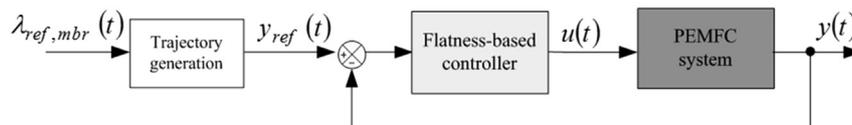


Fig. 1 – Overall control scheme.

Table 1 – Flatness-based controller parameters.

Symbols	Variables	Values
M_{O_2}	Air molar mass	$32 \times 10^{-3} \text{ kg mol}^{-1}$
M_{N_2}	Nitrogen molar mass	$28 \times 10^{-3} \text{ kg mol}^{-1}$
M_v	Vapor molar mass	$18 \times 10^{-3} \text{ kg mol}^{-1}$
R_{O_2}	Oxygen gas constant	$259.8 \text{ J kg}^{-1} \text{ K}^{-1}$
R_{N_2}	Nitrogen gas constant	$296.8 \text{ J kg}^{-1} \text{ K}^{-1}$
R_v	Vapor gas constant	$461.5 \text{ J kg}^{-1} \text{ K}^{-1}$
R_a	Air gas constant	$286.9 \text{ J kg}^{-1} \text{ K}^{-1}$
y_{O_2}	Molar fraction of oxygen in dry air	0.21
\mathcal{L}_{ca}	Orifice flow constant	$2.5 \times 10^{-3} \text{ kg s}^{-1} \text{ kPa}^{-1}$
n	Number of cells	100
F	Faraday constant	96,485 °C
$P_{ca,in}$	Pressure of inlet gas at the cathode	150 kPa
P_{rm}	Pressure of return manifold	140 kPa
V_{ca}	Volume cathode	0.01 m ³
$\phi_{ca,in}$	Relative humidity of inlet gas at the cathode	80%
k_1	Controller parameter	1.3×10^3

In this study, to achieve water management improvement, y_{ref} is calculated based on the desired average membrane water content value $\lambda_{ref, mbr}$, using the following procedure. First, the average water activity a_m is calculated based on [23]:

$$36a_m^3 - 39.85a_m^2 + 17.81a_m + 0.043 - \lambda_{ref, mbr} = 0 \quad (11)$$

The average water activity a_m is the real solution of the above equation. Indeed, since $\lambda_{ref, mbr}$ is a real positive number, it can be demonstrated that Eq (11) admits one real solution and a complex conjugate pair of solutions.

Cathode water activity a_{ca} is then obtained from a_m and the anode water activity $a_a = P_{v,a}/P_{sat}^{T_{a,in}}$:

$$a_{ca} = 2a_m - a_a \quad (12)$$

where $P_{v,a}$ and $T_{a,in}$ denote the vapor partial pressure and the inlet gas temperature at the anode, respectively.

Eventually, the desired mass of water is obtained by:

$$y_{ref} = V_{ca} a_{ca} P_{sat}^{T_{ca,in}} / (R_v T_{st}) = f(\lambda_{ref, mbr}) \quad (13)$$

In the present work, the desired output y_{ref} is calculated according to the desired average membrane water content. As

a result, the membrane water content λ_{mbr} is indirectly controlled by following a specific trajectory of mass of water.

Note that m_{O_2} , m_{N_2} , $m_{v,mbr}$ and their derivatives are not considered here as state variables, but as known disturbances.

The overall control scheme of the water management is clearly illustrated in Fig. 1.

Simulation-based results

In this section, the performance of the proposed control strategy is investigated in simulation environment. In this aim, a mechanistic model, presented and experimentally validated in Refs. [24], is used to simulate a 10 kW PEMFC (CEA LITEN Grenoble, France) presented in Ref. [25]. Both process simulator and flatness-based controller are implemented into the Matlab® environment.

Here, three scenarios are considered to exemplify the controller performance. The first scenario illustrates the tracking capability, whereas the second one evaluates the disturbances rejection ability. The third scenario is designed to assess the controller performance in terms of robustness

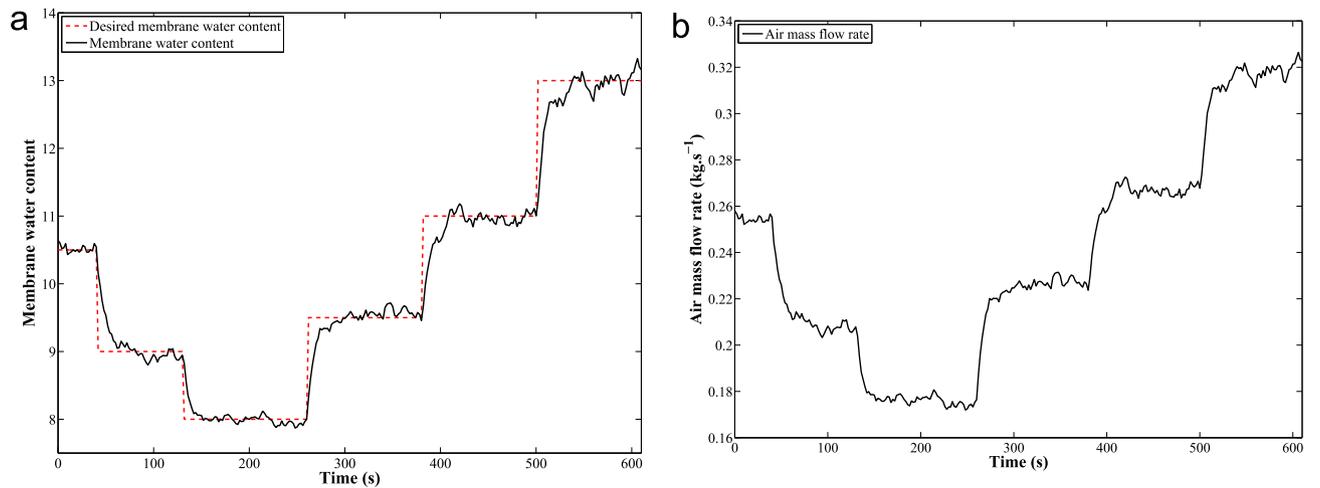


Fig. 2 – Tracking capability: (a) membrane water content; (b) air flow rate.

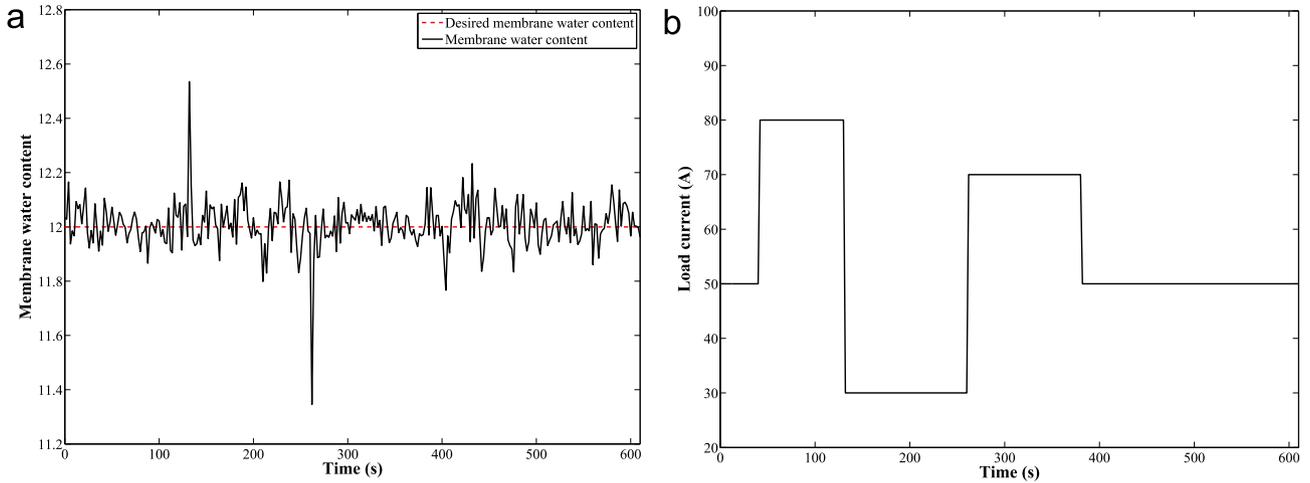


Fig. 3 – Disturbances rejection ability: (a) membrane water content; (b) measured disturbance.

against parameter uncertainties, which are unavoidable in real-world conditions. In the following, to be as close as possible to the experimental context, a white Gaussian noise with a signal to noise ratio of 30 dB is added to the outputs of the process simulator. It is obvious that to ensure optimal performance and safety operations, others variables such as stack temperature have to be regulated. It is thus assumed that all these variables are properly controlled. Flatness-based controller parameters are depicted in Table 1. In the sequel, the inlet air mass flow rate is the manipulated variable, the current is considered as a measured disturbance, and the sampling period is set to 1 s.

First case scenario

As illustrated in Fig. 2, the controller tracks efficiently the desired membrane water content $\lambda_{ref, mbr}$, while offering an entirely suitable dynamic for the manipulated variable. Indeed, with a mean absolute error of 0.15 the controller demonstrates excellent tracking capability, which allows to manage efficiently the membrane water content λ_{mbr} . It can be pointed out that the inlet air mass flow rate computed by the flatness-based controller is affected by the noise. This is due to

the fact that the control law is directly deduced from the model of the process. In practice, this limitation is easily overcome either filtering on-line the measurements or directly the control action.

Note that in this control scenario, the load current is set to 50 A and maintained constant.

Second case scenario

In this case, disturbances rejection ability of the controller is exemplified performing a set of step changes on the load current. To ensure that the flatness-based controller performs accurately regardless of the power level, load current steps are chosen to cover the whole operating conditions. The flatness-based controller exhibits good disturbances rejection capability (Fig. 3). Indeed, set-point tracking errors induced by load current steps are swiftly compensated.

Third case scenario

To appraise the robustness of the controller against plant-model mismatch, a modeling error is introduced between

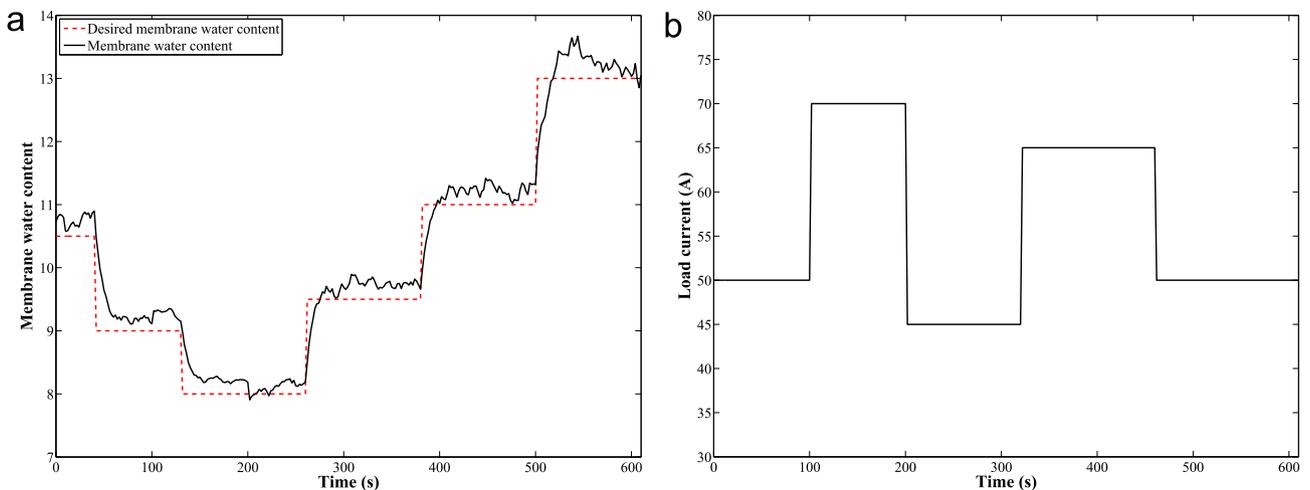


Fig. 4 – Controller performance in realistic conditions: (a) membrane water content; (b) measured disturbance.

the simulator and the flatness-based controller. To this end, the orifice flow constant \mathcal{K}_{ca} is increased by 20% in the control laws. The orifice constant used in the control law is taken to be $3 \times 10^{-3} \text{ kg s}^{-1} \text{ kPa}^{-1}$, whereas the one used in the process simulator is set to be $2.5 \times 10^{-3} \text{ kg s}^{-1} \text{ kPa}^{-1}$.

With a mean absolute error of 0.26 the proposed control strategy exhibits excellent robustness against parameter uncertainties, even in the presence of measurement noise. Indeed, the controller tracks efficiently the desired membrane water content, while compensating the process disturbances (Fig. 4). It can be noted that the modeling error involves a small steady-state error. In the present case, regarding the control objective, the accuracy remains entirely satisfactory. However, if required, this error can be further reduced by introducing an integral term in the closed-loop control law [26–28].

It is important to note that in addition to the presented examples, extensive simulation analysis has been conducted to assess the controller performance. In each case, highly satisfactory results have been obtained, in terms of tracking capability, disturbances rejection ability and robustness against parameter uncertainties and measurement noise.

Conclusions

In this study, a novel approach based on Differential Flatness Theory, has been proposed in views of improving PEMFC water management. This approach has been designed to ensure optimal PEMFC efficiency, while avoiding flooding or membrane drying. In this aim, a flatness-based controller has been developed to regulate the membrane humidity using the air mass flow rate as manipulated variable. The efficiency and the relevance of this strategy have been assessed in simulation environment through several control scenarios for a 10 kW PEMFC system. In each case, the proposed strategy demonstrated highly satisfactory results, since it successfully managed the PEMFC water content. Moreover, the controller exhibited excellent tracking capability, and disturbances rejection ability, even in presence of parameter uncertainties and measurement noise.

The proposed flatness-based approach appears to be a very promising strategy. Indeed, a proper membrane humidity regulation can not only improve the PEMFC efficiency, but may also extend its lifetime by preventing irreversible damages.

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