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SCORE ANALYZER: AUTOMATICALLY DETERMINING SCORES DIFFICULTY LEVEL FOR INSTRUMENTAL E-LEARNING

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ABSTRACT

Nowadays, huge sheet music collections exist on the Web, allowing people to access public domain scores for free. However, beginners may be lost in finding a score appropriate to their instrument level, and should often rely on themselves to start out on the chosen piece. In this instrumental e-Learning context, we propose a Score Analyzer prototype in order to automatically extract the difficulty level of a MusicXML piece and suggest advice thanks to a Musical Sign Base (MSB). To do so, we first review methods related to score performance information retrieval. We then identify seven criteria to characterize technical instrumental difficulties and propose methods to extract them from a MusicXML score. The relevance of these criteria is then evaluated through a Principal Components Analysis and compared to human estimations. Lastly we discuss the integration of this work to @-MUSE, a collaborative score annotation platform based on multimedia contents indexation.

1. INTRODUCTION

In the context of knowledge transmission, musical know-how presents specific features to be efficiently preserved and shared. Indeed, to play correctly and nicely an instrument, one should at the same time acquire physical (gestures, hands position, listening) and intellectual (music theory, score reading) skills. As such, conceiving a service to preserve, transmit and share musical know-how is a complex issue, as we deal with both music hearing faculties and artistic gestures production.

While more and more instrumental e-Learning services are proposed to music amateurs (Garage Band¹, Song2See², iScore³), few of them aims at sharing instrumental know-how on a large scale. Therefore, we propose to build a Musical Sign Base (MSB), grounded on the

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Sign Management methodology [1], in order to collect annotated performances (personal interpretations or stances) each related to a given musical work (class). This base can be used to compare various performances from music experts or students, and also to dynamically build new music lessons from the available content. To allow musicians to feed this base, we designed a collaborative score annotation platform: @-MUSE (@notation platform for MUSical Education). It allows users to illustrate abstract scores (notation) with multimedia content depicting advices, exercises or questions related to the piece (annotation) [2]. However, learners may want to be guided in their choice of a new piece to learn, and to obtain rapidly some starting recommendations to begin learning it on appropriate bases, before any teacher can annotate the piece. That is why, annotations created previously on similar pieces can be useful in this frame in order to depict basic information on the new piece.

To do so, we present in this paper a Score Analyzer prototype in order to automatically identify remarkable parts in a musical piece, from a performer viewpoint. For the time being, we choose to concentrate on the piano for several reasons: the authors are pianists and work in collaboration with piano and guitar experts from music conservatories, but also, the piano repertoire is extremely rich, both historically and technically. Indeed, we want our system to be able to manage not only basic know-how, but also advanced one, on virtuoso instrumental works.

In the first part of this work, we explore existing methods to automatically extract musicological and technical information from a digital score. For this knowledge to be relevant to performers, we base this study on the needs of a pianist who would discover a new piece, following the process generally used by piano teachers to introduce a new work to their students. We then propose seven criteria to characterize technical instrumental difficulties and give methods to extract them from a MusicXML score. The relevance of these criteria is then evaluated through a Principal Components Analysis (PCA) and compared to human estimations. Lastly we discuss the integration of this work to @-MUSE, our collaborative score annotation platform.

¹ <http://www.apple.com/fr/ilife/garageband/>

² <http://www.songquito.com/index.php/en/>

³ <http://rcmusic.ca/iscore-home-page>

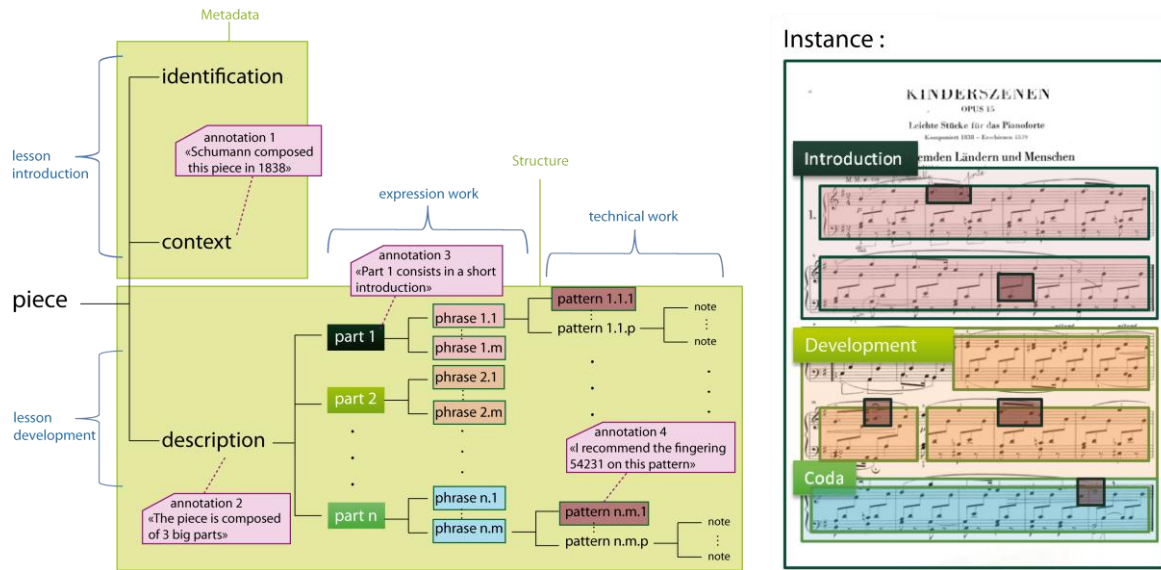


Figure 1. Generic model for musical pieces descriptive logics

2. MUSIC EDUCATION AND ARTIFICIAL INTELLIGENCE

The learning of an instrument generally consists in assimilating a basic repertoire to progress while enjoying playing real artistic compositions instead of only repeating scales mechanically, which can be boring and demotivating. Most of these technical points are directly dealt in the context of the considered pieces. This is why it is essential to select an appropriate corpus for the learner, and to quickly detect remarkable technical points in order to assimilate them, and then concentrate on higher-level considerations, such as expression and musicality. Pointing such features is generally the job of the teacher, until the learner is able to detect them by himself (self-regulation). In the frame of the @-MUSE project, our aim is to assist musicians in this procedure using descriptive logics adapted to each piece genre (baroque, classical, romantic, etc). Figure 1 details a generic model to instance each descriptive logic. It is derived from how teachers introduce new pieces to their students [4]. To extract the different necessary information, we use the standard MusicXML format [3] which describes scores logically, staff by staff, measure by measure, and lastly note by note (Figure 2).

As shown on Figure 1, the first step in our model consists in placing the musical work in its context (composer, period, form metadata). In our frame, it can

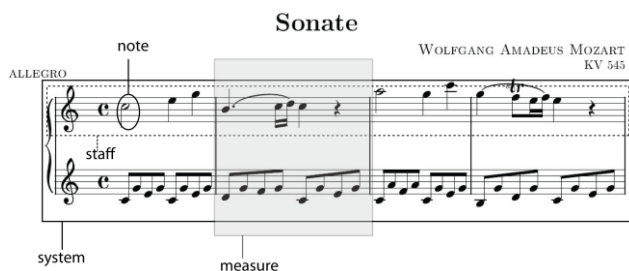


Figure 2. Musical score logical structure

be done using metadata such as title or composer, present in the MusicXML file. In addition, specialized music web services such as MusicBrainz¹ or Last.fm² can be queried to obtain more metadata to illustrate the piece (for instance, a portrait and biography of the composer, or an indication about the piece style). Several performances of the piece can be retrieved from video sharing websites in order to get a glimpse of how the piece should sound.

The second step is to analyze the global form of the piece. Most information about it exists within the piece title (i.e.: Sonata, Fugue, etc.). The challenge is thus to detect the main parts of the piece which characterize its form (i.e.: Introduction, part 1, part 2, Coda). Indeed, grasping its structure is essential to performers, as each part may sound totally differently (especially on advanced pieces). In our frame, this also enables a better indexation for annotations. To achieve that goal, we propose to rely on some of the characteristic tags within the MusicXML file. Indeed, score symbols such as direction texts (e.g. "meno mosso"), tempo and key modifications, double bars generally indicate the beginning of a new part within the piece. While this method seems quite "naive", it gives acceptable results most of the time. Some exceptions may occur, especially on contemporary pieces, which present unconventional structures.

After indicating main parts of the piece, the teacher generally brings the attention of the learner on the remarkable rhythmic or harmonic patterns the piece is build on (if any), leading to more technical and detailed practice. In our work, discovering predefined patterns such as scales, arpeggios or trills may be done using a memory window of successive intervals. Indeed, scales will correspond to sequences of ascendant or descendant seconds, arpeggios to sequences of triples, etc. Each detected pattern can then be linked to a generic annotation explaining how to work on it. However, detecting more complex and

¹ <http://musicbrainz.org>, visited on the 10/04/2012.

² <http://www.last.fm>, visited on the 10/04/2012.

non-determined patterns remains a challenge, as it does not only involve rhythms and pitch features, but also polyphonic ones. Moreover, it does not present a unified definition of “similarity”. Two fragments can be considered as “similar”, without having the same pitches, but by possessing similar intervals (transposition). Several works exist on Musical Pattern Discovery. Among them, [5] presents a method based on time windows and define different types of patterns (abstract patterns, prefixes, patterns network). Still, each suggestion given by our system calls for a validation by a music professional.

In order to semantically annotate the detected structures, we need a musical form ontology. While the Music Ontology [6] is particularly fitted to the music industry, it lacks some concepts to be effective in music education. More specialized ontologies exist, such as the Symbolic Music Ontology (allowing to manipulate Voices and Motifs concepts), the Chord Ontology or the Neuma ontology (for Gregorian Music) [7], however, a real form taxonomy has yet to be built to manage the manipulation of concepts such as Sonata, Fugue, Theme or Coda.

The last step of our introduction lesson is to underline specific difficulties of the piece. This will allow us to

both specify the global level of the piece, and to detect its technical difficulties measure by measure. To do so, we propose in what follows seven criteria to evaluate a piano piece difficulty.

3. CRITERIA DEFINITION AND RETRIEVAL

In Table 1, we propose seven criteria affecting the level of a piece for the piano and detail how they can be estimated from a MusicXML file. These criteria were defined on the base of pianists experiences, both professionals and amateurs. They may be applied to other instruments with some adaptations (see Instruments column in Table 1). Globally, a piano piece difficulty depends on its tempo, its fingering, its required hand displacements, as well as its harmonic, rhythmic and polyphonic features. Although we define each criteria separately, they affect each other in a complex manner. In particular, fingerings remain hard to extract from a score, as most MusicXML files do not contain this information. Indeed, while other criteria reside in the basic notation layer (notes pitch and duration), the fingering is from the annotation layer and directed at humans only (human performance information).






Performance difficulty criterion	Definition	MusicXML implementation	Instruments
Playing speed	The required fingers velocity to play the piece. Depends on the tempo and the shortest significant note value (i.e. a piece presenting a high tempo may contain only long values, and conversely, a piece with a low tempo may contain groups of short notes thus increasing the required fingers agility for the players)	<code><note><type></code> elements Tempo attribute in <code><sound></code> element	All
Fingering	Fingering: choice of finger and hand position on various instruments. Different notations exist according to the instrument. (ex: in piano: 1 = thumb, 2 = index finger, 3 = middle finger, etc.) Cost functions are used on intervals to extract the general fingering difficulty level See [8][9] for more detail.	<code><fingering></code> element within each <code><note></code> element 	All, requires adaptations in constraints and costs functions (some instruments do not use thumbs)
Hand Displacement	Ratio of hands displacements greater than an octave (12 semitones). Depends on the duration of the interval: if the duration exceeds 2 beats (i.e. 2 quarters in 4/4, 2 eights in 6/8), the displacements is not considered as difficult. The difficulty degree of the displacement evolves with its size (in pitch), its duration and its fingering	Combined <code><note></code> elements where <code><pitch></code> gap > 12 and <code><duration></code> gap < 2 beats 	All, requires adaptations depending on the instrument morphology
Polyphony	Chords ratio (aggregate of musical pitches simultaneously attacked) Polyphonic difficulties may increase with the number of notes played at the same time and their fingerings. Simultaneous voices (in a Fugue for instance) constitute special cases of polyphonic difficulties to treat.	<code><chord></code> element 	All (except for monophonic instruments, such as the flute)
Harmony	Ratio of differences from the piece main tonality. Characterized by the amount of accidental alterations.	<code><alter></code> and <code><accidental></code> elements 	All
Irregular Rhythm	Ratio of irregular polyrhythms (simultaneous sounding of two or more independent rhythms). Example: synchronizing a triplets over duplets	<code><time-modification></code> element 	All (except for monophonic instruments)
Length	The number of pages of the score. May also be measured in bars number to avoid dependency to the page layout.	new-page attributes or <code><measure></code> elements	All

Table 1. Performance difficulty criteria in piano practice

Several works present methods to automatically deduce fingerings on a given musical extract for piano ([8][9][10]). Most of them are based on dynamic programming. All possible fingers combinations are generated and evaluated, thanks to cost functions. The latter are determined by kinematic considerations. Some functions, even consider the player's hand size to adjust its results. Then, expensive (in term of effort) combinations are suppressed until only one remains, which will be displayed as the resulting fingering. While the result often differs from a fingering determined by a human expert, it remains largely playable and exploitable in the frame of an educational usage. However, few algorithms can process polyphonic extracts, and many other cases are ignored (i.e., left hand, finger substitutions, black and white keys alternation).

Even if more work is needed on this issue, the use of cost functions remains relevant as it is close from the process humans implicitly apply while working on a musical piece. Therefore, we use this method in our Score Analyzer prototype to translate extracted criteria into difficulty indicators (see part 5). But to do so, we need to study how our criteria discriminate a corpus of piano pieces, both objectively (through a components analysis) and subjectively (based on pianists experience).

4. PIANO SCORES CORPUS CLUSTERING

To study how our criteria discriminate scores, we realized a PCA on a sample of fifty piano pieces (Figure 5). The pieces were selected to be representative of a classical piano cursus in a French Music Conservatory. Most pieces concern intermediate to advanced players, fewer target beginners and virtuosi. Most MusicXML files were retrieved from online music notation communities such as MuseScore.com, Noteflight or the Werner Icking Music Archives. Some were generated from PDF files using the SmartScore™ OCR software.

The criteria defined in Table 1 were extracted on each piece. Displacements, chords and harmonic characteristics are distinguished whether they occur on the right (RH) or the left hand (LH). Fingerings were not exploited for the time being as work is in progress to deduce them from MusicXML files (see part 3). Our analysis thus counts 9 numeric variables (Figure 4), and 1 nominal variable (composer). Each ratio is calculated on the base of the total number of notes (e.g. harmonic criteria), or the total number of hands positions (e.g. displacements, chords) within the piece. A displacement is thus defined as a pair composed of two successive hand positions.

A correlations study (Figure 3) points out some links between variables. Some are musically natural (i.e. harmonyLH and harmonyRH, harmonic characteristics concern both hands). We also note a strong correlation (81%) between chordsLH and displacementsLH. This value could characterize accompaniments presenting an alternation of a low-pitched bass and a middle or high-pitched chord, thus inducing regular large displacements and

chords at the left hand (ragtime, waltz). Lastly, the piece length can be linked to its playing speed, which characterizes advanced and virtuosi works, demanding an important fingers velocity on a long duration (stamina).

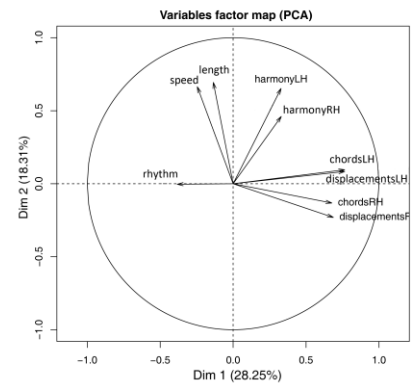


Figure 3. Variables correlation map

The PCA then gives an optimal projection of each piece in the 2D space of the first principal components. Figure 5 presents this projection as well as the three classes detected by the analysis. This clustering was realized through a hierarchical clustering using the Ward's method [11] on the first few principal components. The resulting tree is then cut according to its corresponding indices, in order to find an appropriate number of clusters. Lastly, this clustering is consolidated using a k-means algorithm. The first interpretation of these three classes validates the relevance of our criteria to estimate the difficulty level of a piano piece. Indeed, we notice that at least two of the classes naturally regroup pieces according to their level (class 1 and 2). A further observation backed by a Student test (variable means comparisons between the whole population and the clusters) gives a better interpretation of the classes. Class 1 mostly regroups pieces addressed to beginners (*Kinderszenen*, Schumann's *Choral*) and to intermediate musicians (Bach's *Invention*, *Sonatines*). The Student test confirms this tendency, as most variables remain below average for this class: few chords, displacements and pages, simple harmonies (C major or A minor). Yet, the tempo remains lively. Rhythmic difficulties are noticeable on intermediate pieces. They generally feature characteristic rhyth-

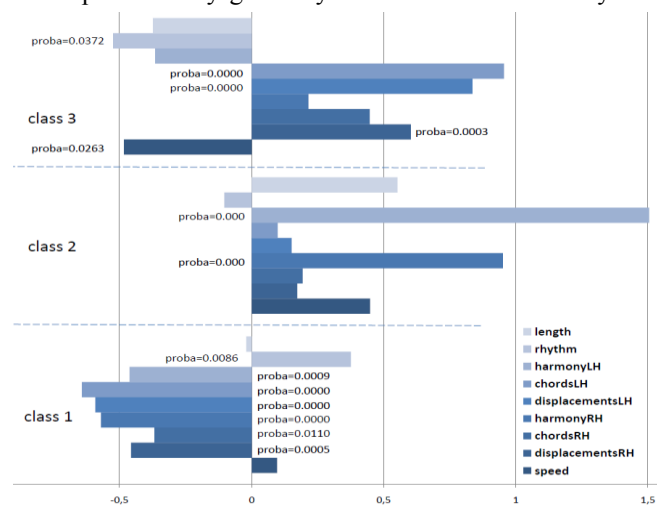


Figure 4. Student test (means comparison)

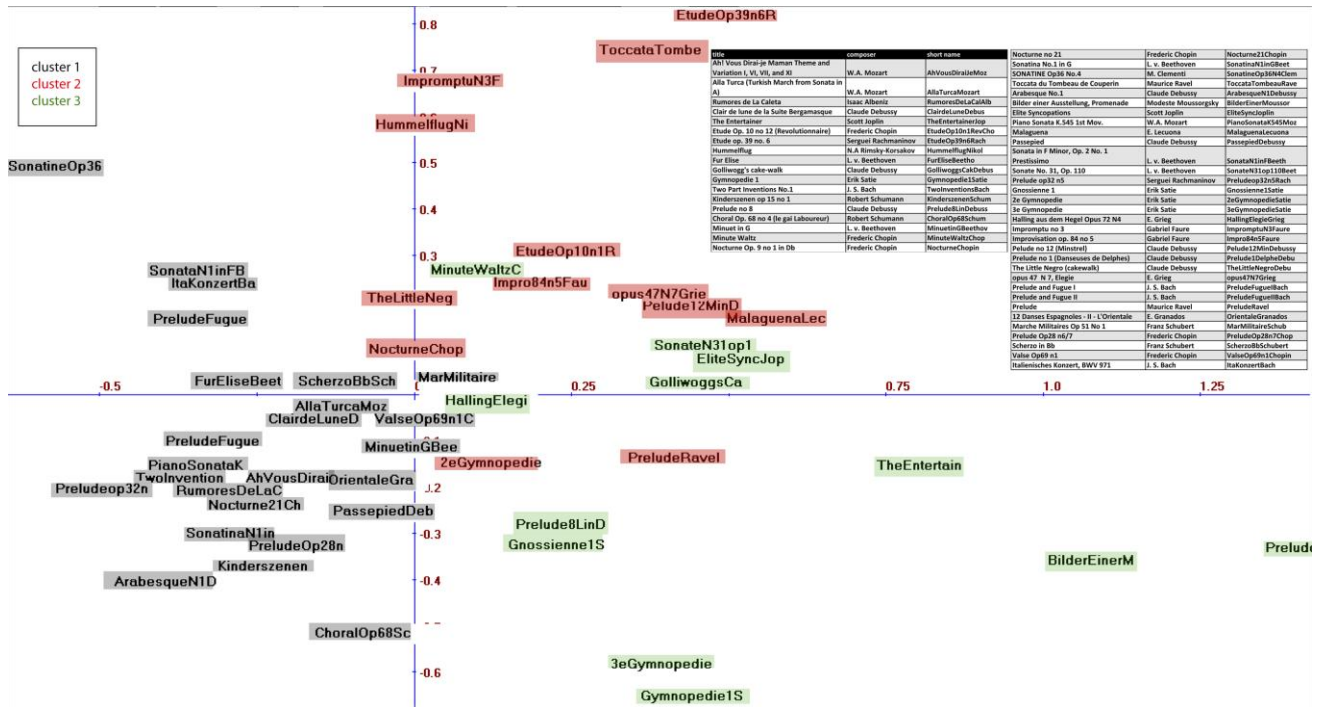


Figure 5. Individuals projection on the PCA first two axes and corpus details

mic patterns which constitute interesting educational material (e.g. 1st Arabesque by Debussy). Class 2 contains advanced to virtuoso works (Chopin’s Etude, Ravel’s Toccata), featuring a vivid tempo, large and numerous displacements on the keyboard, a complex harmony and many chords. We also note some borderline individuals (The Little Negro by Debussy, or the 2nd Gymnopédie by Satie), which could be considered as beginner pieces but still present uncommon harmonic and rhythmic structures, thus being hard to classify objectively. Class 3 seems to regroup pieces featuring a left hand playing a “bass+chord” accompaniment (ragtime, waltz, cakewalk). The level of most pieces is intermediate. Indeed, the Student test indicates that despite the high ratio of displacements and chords, the low tempo and the simplicity of the harmonies compensate for it. As such, this particular class is also representative of specific musical genres. This clustering serves as a complement to the “bounds” approach used in Score Analyzer.

5. SCORE ANALYZER PROTOTYPE

The criteria presented in the previous sections have been implemented in a Web application called Score Analyzer¹ (SA). This module is integrated to the @-MUSE platform as a Web service in order to automatically evaluate a piece level and identify its difficult parts. The SA engine takes any well-formed MusicXML file as input and parses it to extract knowledge exploitable from a performer point of view. Following the scheme we detailed previously (Figure 1), the context of the piece is briefly analyzed (title, composer) and a few statistics are

displayed. Then, main parts of the piece are identified, and lastly, difficulty estimations are given for each criterion, using a mark from 1 (beginner/easy) to 4 (virtuoso). A mean is also calculated to give a global appreciation of the piece difficulty. This allows a better readability of the outputs for musicians. For each criterion, bounds were defined with the help of teachers: for instance, a chord ratio under 10% corresponds to the mark 1, while a displacement ratio above 20% corresponds to a 4. These bounds determination was transparent for teachers, as they were simply asked to rate each criteria from 1 to 4 on a training corpus. The given marks were then correlated to the ratio extracted on each piece, in order to calibrate average bounds corresponding to the difficulty levels felt by musicians. Thus, we notice that most of the criteria do not have a linear distribution, which constitutes a pianistic reality. The synchronization between both hands is also taken into account. For instance, if each hand obtains a mark of 2 for the displacements criterion, then the global difficulty mark for this criterion will be 3, as synchronizing both hands will create an additional difficulty.

As such, we define this method as “semi-objective”. Indeed, score level estimation can never be a totally objective task: players will judge a piece differently according to their taste, level or background. Therefore, we use two distinct methods to validate SA estimations. The first one consists in confronting it to the clustering obtained through the PCA described in the previous part. This is the “objective” validation. The second one simply consists in confronting SA results to pianists estimations (“subjective” validation). To facilitate the comparisons, we merged advanced and virtuosi pieces into the same class within SA. The contingencies table (Table 2) allows to better visualize the differences between the PCA and

<http://e-piano.univ-reunion.fr/tests/ScoreAnalyser/readScore.php>, visited on the 05/06/2012, beta version.

SA marks \ PCA marks	1	2	3
1	TwoInventionsBach KinderszenenSchum ChoralOp68Schum	SonataN1inGBeeth PianoSonataK545Moz	
2	AhVousDiraiJeMoz AllaTurcaMozart RumoresDeLaCalAlb ClairdeLuneDebus FurEliseBeetho MinuetinGBeethov Nocturne21Chopin SonatineOp36N4Clem ArabesqueN1Debussy PassepiedDebussy	PreludeOp32n5Rach PreludeFugueIIIBach OrientaleGranados PreludeOp28n7Chop Scherzo8bSchubert ValseOp69n1Chopin MarMilitaireSchub ItaKonzertBach	TheEntertainerJop GolliwoggsCakDebus Gymnopedie1Satie BilderEinerMoussor EliteSynclopin Grossienne1Satie 3eGymnopedieSatie HallingElegieGrieg Prelude1DelpheDebu
3	SonataN1inFBeeth	Prelude8LinDebuss MinuteWaltzChop SonateN31op110Beeth	EtudeOp10n1RevCho EtudeOp39n6Rach NocturneChopin Toccatatombearuave MalaguenaLecuona ImromptuN3Faure Improb4n5Faure

Table 2. Contingencies table between SA and PCA marks

teachers marks \ SA marks	1	2	3
1	TwoInventionsBach KinderszenenSchum ChoralOp68Schum SonataN1inGBeeth	PianoSonataK545Moz	
2	2eGymnopedieSatie MinuetinGBeethov 3eGymnopedieSatie PreludeFugueIIIBach PreludeOp28n7Chop ValseOp69n1Chopin TheLittleNegroDebu	TheEntertainerJop GolliwoggsCakDebus Gymnopedie1Satie BilderEinerMoussor EliteSynclopin Grossienne1Satie Prelude1DelpheDebu HallingElegieGrieg Prelude8LinDebuss MinuteWaltzChop SonateN31op110Beeth	FurEliseBeetho Nocturne21Chopin SonatineOp36N4Clem ArabesqueN1Debussy PreludeFugueIIIBach OrientaleGranados Scherzo8bSchubert MarMilitaireSchub opus47N7Grieg PreludeRavel ItaKonzertBach
3			EtudeOp10n1RevCho EtudeOp39n6Rach NocturneChopin Toccatatombearuave MalaguenaLecuona ImromptuN3Faure SonataN1inFBeeth

Table 3. Contingencies table between SA and teachers marks
Score Analyzer's results. While they seem numerous, only one is a major disagreement (3/1 marks on Beethoven Sonata in F). The other distinctions, especially the intermediate/beginners ones, may be due to the fact that humans balance criteria whereas the PCA considers each of them of equal importance. Therefore, we noticed that for pianists, an increase of the displacement ratio raises the piece level much faster than other criteria. Moreover, as stated in the previous part, the clustering given by the PCA is also affected by the musical genre of the piece. Humans do not tend to be affected by this metadata, even if some genres are naturally associated with higher levels (i.e. impressionist or contemporary music).

For the "subjective" evaluation, we asked three piano teachers to estimate the difficulty level of each piece by attributing it a mark between 1 and 3. No criteria were imposed. When opinions differ, the final mark is picked according to the majority. The results given in Table 3 show a better correspondence between SA estimations and human ones, which reinforces the "bounds" method defined previously. The main difference consists in underestimations from SA, especially on advanced pieces. Indeed, pianists also take expression and musicality difficulties into account, while our system only consider technical difficulties. Therefore, this study leads us to pursue our work by expanding the set of criteria to improve our estimations.

6. CONCLUSION

In this paper, we proposed an automatic Score Analyzer to determine the difficulty level of piano pieces. This prototype is based on seven criteria characterizing technical features of a piano piece: playing speed, fingerings, hands

displacements, polyphony, harmony, rhythm and length. We thus proposed methods to extract these criteria from a MusicXML scores, and realized a PCA to validate them. This analysis permitted to establish three classes among a corpus of fifty selected piano pieces. These classes were then confronted to Score Analyzer estimations, which are tuned according to piano teachers expertise.

Improvements on this work include the integration of fingering related difficulties, but also the adaptation to students levels. Indeed, the sense of difficulty within a musical work is mostly dependent from the musician's background. We thus imagine a weighting system to personalize our analysis. We also intend to implement local analysis (by measures) in order to identify specific difficult parts. The criteria decomposition would then allow to extract the main cause of the difficulty and thus link it to an annotation created on the @-MUSE platform. Other perspectives include integration of "expressive" criteria (emotions, nuances, rubato, attacks), as well as adaptations and tests on scores for different instruments.

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