DYNAMIC MODELLING AND ELEMENTS OF VALIDATION OF SOLAR EVACUATED TUBE COLLECTORS

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ABSTRACT

This paper deals with the dynamical modelling of a solar evacuated tube collector under variable weather conditions. After describing the mathematical model, the paper presents elements of validation. In a first step, the theorical model is validated against the experimental results. Then a parametric sensitivity analysis is applied to the model. Such a study is very interesting for modellers in order to determine the relative importance and the nature of the effect of the parameters. It is important so as to improve the model by controlling this paramaters or accurately measuring it.

1. INTRODUCTION

Solar buildings technologies use the clean power of the sun to heat cool and power buildings. The starting point of most active solar energy systems is solar collectors. Solar cooling is one of the most attractive applications, the incidence of solar energy and cooling requirements are indeed approximately in phase. Evacuated tube collectors are particularly appropriate for this application. These collectors perform well in both direct and diffuse solar radiation and offer the advantage that they work efficiently with high absorber temperatures.
The final aim of this study is the modelling of a global solar cooling system from the heat production to the cooling supply coupled with the building loads. The first step of the work presented in this paper is the modelling of the heat production provided by solar collectors.

The litterature contain numerous works on the modelling of solar collectors. These models developed have differents levels of complexity. Usually, solar collectors are described by stationnary models, considering the collector working under steady-state conditions. These approach are generally based on the work of Klein (Klein et al., 1974). The principal advantages of this type of models are to be simple and have an high speed calculations.However, it is well known that large overprediction may occur by stationnary model, (Isakson and Eriksson, 1991). When coupling a stationnary model with components whose behaviour depend on time cause additionnal

errors to energy yield predictions, (Schnieders, 1997).

A dynamic approach is more interesting in several cases: control strategies, dynamic testing procedures, coupling with others elements. Particulary, predict the behaviour of collectors for a time step much than hourly step, a dynamic modelling bring more informations concerning the collector. The principal idea underlying the work reported here is the following. The collector is modelling under a short time step in order take into account the variation of the meteorogical parameters. This level of description allows us to apply sensitivity analysis and understand wich paramaters have a significant influence on the oulet temperature of the collector. In the following, all physical phenomena are investigated separately to describe the model. The dynamic behaviour of the model is verified thanks to numerical tests and measures comparisons.

2. DYNAMIC DESCRIPTION OF THE COLLECTOR MODEL

The model developed corresponds to direct flow collector. It is not appropriated to vacuum tube used specific fluid in heat pipes to heat the collector inlet fluid by an exchanger. The type of solar collector modeled consists of six vacuum tubes. The heat transfer fluid flows in a copper U-tube which is welded to a narrow flat absorber. Thus, the inlet and the outlet are at the same end of the evacuated tube.

In order to model the evacuated tube collector, a number of simplifying assumptions have to be made. Most of these have been previously described by Duffie & Beckmann (Duffie, 1991). Perfect insulation at the edges of the collector is assumed. No heat is supposed to be transported in the fluid moving direction, conduction is neglected. The gradients inside the glass cover and the absorber plate are assumed to be negligible. As the main objective lies on collector dynamical behaviour modelling, the effect of incidence angle is also neglected. As the collector studied is a vacuum tube, free convection inside the glass tube is not taken into account. In the following sections, all heat transfer

occured in the solar collector are defined. Figure 1 present the general description of thermal transfer in the solar collector.
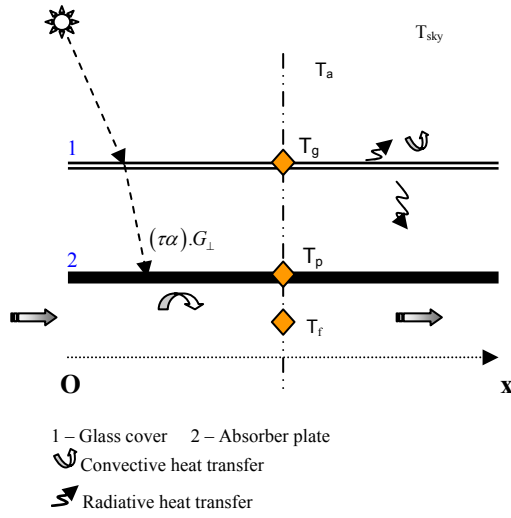

1 – Glass cover   2 – Absorber plate

∿ Convective heat transfer

➹ Radiative heat transfer

Figure 1 General description of the solar collector model.

Each components of the solar collector (the fluid, absorber plate, glass cover) are considered separately. Each element has his own heat capicity.

Collector differential equation system

The starting point of the model is a mathematical description proposed by Kamminga (1985). The model consists on three nodes corresponding to the fluid, the absorber plate and the transparent glass cover. It is considered that the temperature of the fluid is a function of x. The fluid is moving in a single channel with the velocity u, along x-axis.

It results a 3-node collector model given by the following differential equation system :

$$C_g \frac{\partial T_g}{\partial t} = \varepsilon_g \sigma \left(T_{sky}^4 - T_g^4 \right) + h_{g-a}(T_a - T_g) + \varepsilon_g \sigma \left(T_p^4 - T_g^4 \right) \quad (1)$$

$$C_p \frac{\partial T_p}{\partial t} = \tau \alpha G_\perp + \varepsilon_g \sigma (T_g^4 - T_p^4) + h_{f-p}(T_f - T_p) \quad (2)$$

$$C_f \left(\frac{\partial T_f}{\partial t} + u \frac{\partial T_f}{\partial x} \right) = h_{f-p} \left(T_p - T_f \right) \quad (3)$$

The system given by this three equations can be solved using Fourier transform of the time dependent set of differential equations (1) – (3). We have chosen to numerically solve this system using finite difference method. In this case, the collector is defined as single fluid channel, which is divided into N segments. The differential equation system is solved for each segments in the time domain using a 4th order Runge-Kutta method. The final oulet

temperature obtain for segment (x_{i-1}) is the initial or inlet fluid temperature for segment x_i. The final oulet temperature is obtained by connecting the N segments of the collector. As proposed by Henning (1995), the partial differential equation (3) can be written as an ordinary equation using the method of charatesristics, Holland and Liapis (1983). The velocity u of the fluid is assumed to be constant, thus equation (3) becomes :

$$C_f \frac{dT_f}{dt} = h_{f-p} \left(T_p - T_f \right) \quad (4)$$

Finally, the new set of equations can be illustrated with the thermal networks shown in figure 2.
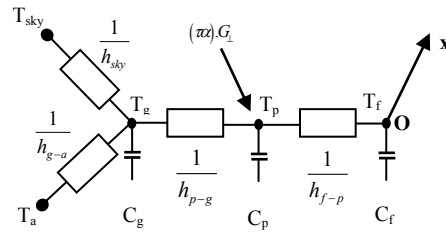

Figure 2 Thermal networks for the 3-node model.

Thus all the components are described in the time domain.

Convection heat transfer from cover due to wind

The convection heat transfer coefficient due to wind from McAdams (1954) is generally assumed:

$$h_w = 5,678 + 3,8.v \quad (5)$$

This correlation are generally used. In the case of vacuum tube collector it is also possible to use the relation from Hilpert (1933) which describes the external fluid flow distribution on a cylinder.

Forced convection heat transfer between absorber and fluid flow

In a laminar flow region, the formalism used has been first described by Colburn (1933)

$$Nu = 0.023 \, Re_D^{\frac{4}{5}} \, Pr^{\frac{1}{3}} \quad (6)$$

In the case of turbulent flow region (Re > 6000), Koo(1999) recommended to use the correlation obtained from the relationship of Gnielinsky :
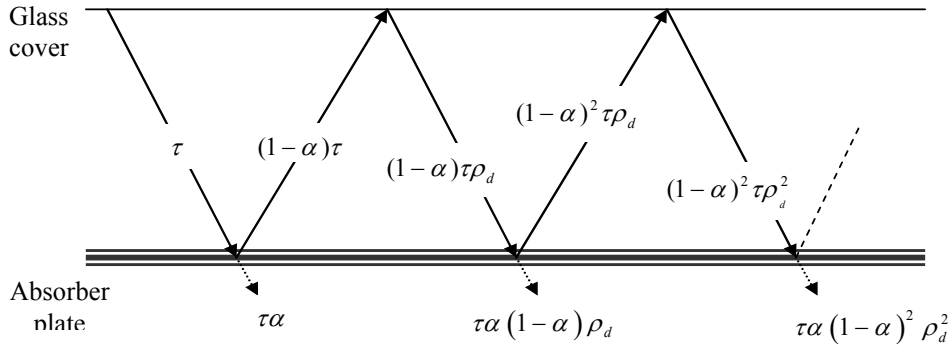
Figure 3 Absorption of solar radiation by absorber plate under cover

$$Nu = \frac{\frac{f}{8}(\text{Re}-1000)\,\text{Pr}}{1 + 12.7\sqrt{\frac{f}{8}\left(\text{Pr}^{\frac{2}{3}}-1\right)}} \qquad (7)$$

The friction factor of Darcy for the above tubes may be obtained from :

$$f = \left(0.0790 \ln \text{Re} - 1.64\right)^{-2} \qquad (8)$$

Long wave radiation transfer between the glass cover and sky

The emissivity of sky is assumed to be equal to 1, thus the long wave flux may be written as:

$$\phi_{LW} = \varepsilon_{sky}\sigma.T_{sky}^4 \qquad (9)$$

For the sky temperature, Boyer (1993) has proposed to use a simple linear relationship depends on air temperature:

$$T_{sky} = T_a - a \qquad (10)$$

The constant a has been defined by optimization, it depends on the place that the simulation is supposed to represent. Garde (1997) suggested to use a = 6, for simulation occur in Reunion.

Solar radiation

The solar radiation heat flux absorbed by the absorber plate surface is defined by:

$$S = (\tau\alpha)G_\perp \qquad (11)$$

As suggested by Duffie (1991), the transmisttance-absorptance product $(\tau\alpha)$ should be thought of as a symbol representing a property of the cover-absorber combination rather than as a product of two properties.

The product $(\tau\alpha)$ is the result of multiple reflection of diffuse radiation so that the fraction of the incident energy finally absorbed is given by:

$$(\tau\alpha) = \tau\alpha\sum_{n=0}^{\infty}\left[(1-\alpha)\rho_d\right]^n = \frac{\tau\alpha}{1-(1-\alpha)\rho_d} \qquad (12)$$

This description is illustrated in figure 3.
The subscript d represents the diffuse radiation in the vacuum tube.

3. EXPERIMENTAL AND SIMULATION

Experimental setup under natural conditions

Data from testing at the University Test Field in Reunion (21°S, 55°E) have been used.
The experiments occur under natural tropical humid conditions. The collector considered was tested on a fixed frame as illustrated by the photograph in figure 4.

Figure 4 The experimental set up

The collector loop has been operated with continuous flow 12 hours a day from sunrise to sunset and in some cases by night. The inlet and outlet temperature are measured. Environmental parameters are also measured as global solar irradiance, air temperature and wind velocity.

The experimental setup is used for *in situ* characterization of the collector according to the European Standard CEN 12975-2. It also constitutes a database for the validation of the solar collector modeling.

The tests performed allow us to have a description of the collector under steady state conditions. Thus, the useful energy and the efficiency of the collector are evaluated.

Collector model predictions

The solar collector model has five input parameters as ambient temperature, solar irradiation, mass flow, inlet temperature and the wind velocity. All these parameters are read from data files. The model calculates the outlet temperature and compares it with the measured outlet.

The database is a minute step data acquisition. The global irradiance in the collector plane and ambient temperature for the examined days are presented in figure 5.
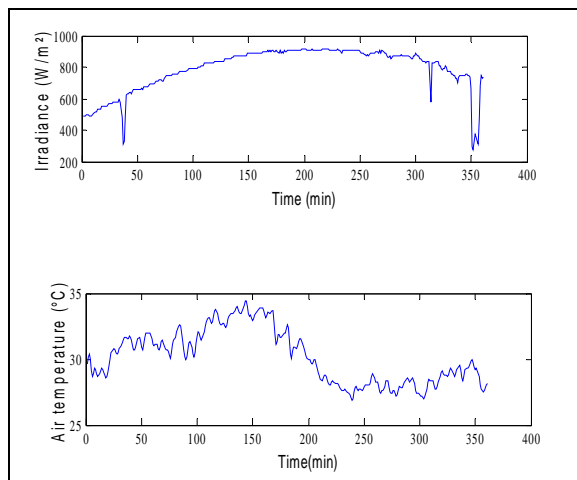

Figure 5 Weather conditions for the examined days.

The results of simulation are drawn on figure 6. The first basic step in validation is a measure – model predictions comparison.

The dynamic model does not much differ in the prediction of the outlet temperature. In the case of solar vacuum tube model computing, the encountered difficulty is the initialization at each time step of the absorber plate temperature. This temperature is not measured, as the collector is under vacuum. Thus at the beginning of the simulation, there is a significant different between measures and model. When simulating, the dynamic behavior of the solar collector, this step have to be consider as

initialization phase. Thus, the values obtained at the beginning of each simulation will not be taken into account.
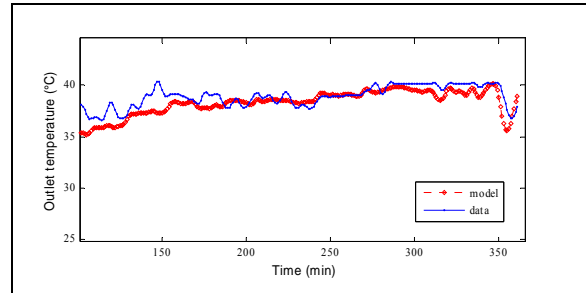

Figure 6 Comparison of measured and modeled outlet temperatures for the vacuum tube collector.

In order to make up to this question, two solutions approach is actually under testing. The first one is a theoretical approach, considering artificial neural networks to model the dynamic behavior of the absorber plate according to metrological data. The other approach is the instrumentation of a second collector, including measures of absorber plate temperature. In this way, it will be possible to initialize all parameters at each time step.

SENSITIVITY ANALYSIS

As defined by Saltelli (1999), the objective of sensitivity analysis (SA) of model output is to ascertain how a given model (numerical or otherwise) depends on its input factors. This way it is also possible to determine if the model does not exhibit unexpectedly strong dependencies upon non influential parameters

This analysis is an important step in the verification and validation of models. Thus SA helps to understand the fundamental mechanisms underlying the behaviour of the model and interactions between the different parameters. There are indeed different types of SA, and a numerous techniques have been developed. In the present work, we move from a method which turns on two consecutives techniques:

- A screening test proposed by Morris (1991), which allows identifying qualitatively, the relative influence of parameters.
- FAST method (acronym of Fourier Amplitude Sensitivity Test) to determine the influence of factor and its nature.

The Method of Morris

The basic idea of this method is to determine, within a reasonable uncertainty, which input parameters could be considered to have a significant influence on the output. The main advantage of this method consists in its short computing time, regarding the

number of simulations. However this statistical method does not allow arranging the parameters in order of influence. In this work, the screening method was used to help us on choosing and focusing on the most important parameters in the second part by applying FAST method.

The Morris method is based on experimental plans that are composed of individually randomized one-factor-at-a-time (OAT). Thus each factor will take only 2 possible values.

Considering a p-dimensional factor vector X of the model, the output is $Y(X_1, ..., X_i, ..., X_p)$.

For a given value of X, the effect of the ith input factor is defined as:

$$d_i^{(1)} = \frac{Y(X_1^{(1)}, ..., X_i^{(1)} + \Delta_i^{(1)}, ... X_p^{(1)}) - Y(X^{(1)})}{\Delta_i^{(1)}} \qquad (13)$$

The first step of the method is the standardization of the factors. If $X_i \in [u_i, l_i]$, thus the standardized expression of X_i is $x_i = (X_i - b_i)/a_i$, with $b_i = (u_i + l_i)/2$ and $a_i = (u_i - l_i)/2$.

Applying this standardization, all parameters are in the same interval for example [0,1].

The second step is the choice of k_i values between [-1,1], so that $x_i.k_i > 2$, here the non linearity of the response is taken into account. Then a randomly selecting values of $x^{(1)}$ allows the calculation of the corresponding $X^{(1)}$ and finally the response $Y^{(1)}$ associated. The parameter X_1 is randomly modified between the k_1 possible values and its effect is evaluated by equation (13).

The procedure is repeated r times. Finally a $d_i^{(r)}$ matrix is obtained:

$$\begin{bmatrix} d_1^{(1)} & d_2^{(1)} & \cdots & d_{p-1}^{(1)} & d_p^{(1)} \\ d_1^{(2)} & d_2^{(2)} & \cdots & d_{p-1}^{(2)} & d_p^{(2)} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d_1^{(r-1)} & \cdots & \cdots & d_{p-1}^{(r-1)} & d_p^{(r-1)} \\ d_1^{(r)} & d_2^{(r)} & \cdots & d_{p-1}^{(r)} & d_p^{(r)} \end{bmatrix}$$

The identification of the relative importance of factors are shown by a graphical analysis of the standard deviation σ_i versus the mean of the effects μ_i due to X_i :

$$\begin{pmatrix} \mu_1 & \cdots & \cdots & \mu_{p-1} & \mu_p \\ \sigma_1 & \cdots & \cdots & \sigma_{p-1} & \sigma_p \end{pmatrix}$$

Figure 7 shows the results obtained on the solar collector model. The total computational cost of the experiment is n=r(p+1) runs.

The data analysis of figure 7 shows that there are four principal parameters which have a significant influence: transmittance-absorbance, absorber surface, capacity of fluid and absorber. The effect of these factors are not correlated and non linear. As we can see on figure 7, the other parameters have a mean effect value which is 0.

Generally, the screening phase is very helpful when the model have a lot of input factors; it allows eliminating parameters that do not have any influence. As we have a short number of parameter for the FAST method all of them will be used. Thus we could verify that the two methods fond the same significant parameters.

The FAST Method

This method is an experimental plan in the spectral domain, developed by Cukier & al. (1973). It allows the computation of the fraction of the variance of a given model output which is due to each input variable.

The guiding idea underlying the method is to apply the ergodic theorem as demonstrated by Weyl (1938). Considering a one output (y) model with p input parameters $y = f(x_1, x_2, ..., x_p)$, the parameter are sampling in their own range of variation. Each parameter x_h includes a periodical function G_h characterized by a frequency w_h. The frequency is the "signature" of the parameter.

Thus, the sampling of the parameter x_h can be expressed by the following formula:

$$x_{h,k} = G_h(\sin(w_h s_k)) \qquad (14)$$

The transformation function G_h is generally chosen to assure a good representation of the wide range of parameters. That means that the variable x_h has to be sampled following a precise given density probability (corresponding to the uncertainty on its value). Mara (2001) proposed to sample the parameters with the following manner:

$$x_{h,k} = x_{h,0} + \delta_h \sin(w_h s_k), \text{ with } s_k = 2\pi k/N_s \qquad (15)$$

Where k represents the simulation number (k = 1 à Ns) , $x_{h,0}$ is the basis value of the parameter h and δh is chosen such as $x_{h,k} \in [x_{h,0} - \delta_h, x_{h,0} + \delta_h]$, Ns is the number of simulations.

The Fourier transform of the output of the model y is calculated and the spectrum is drawn. We identify the frequencies which appear at each step of the graphical analysis. The frequency assigned to each parameter is indexed in table 1. The next step is the identification according to their frequency of the most important parameters.
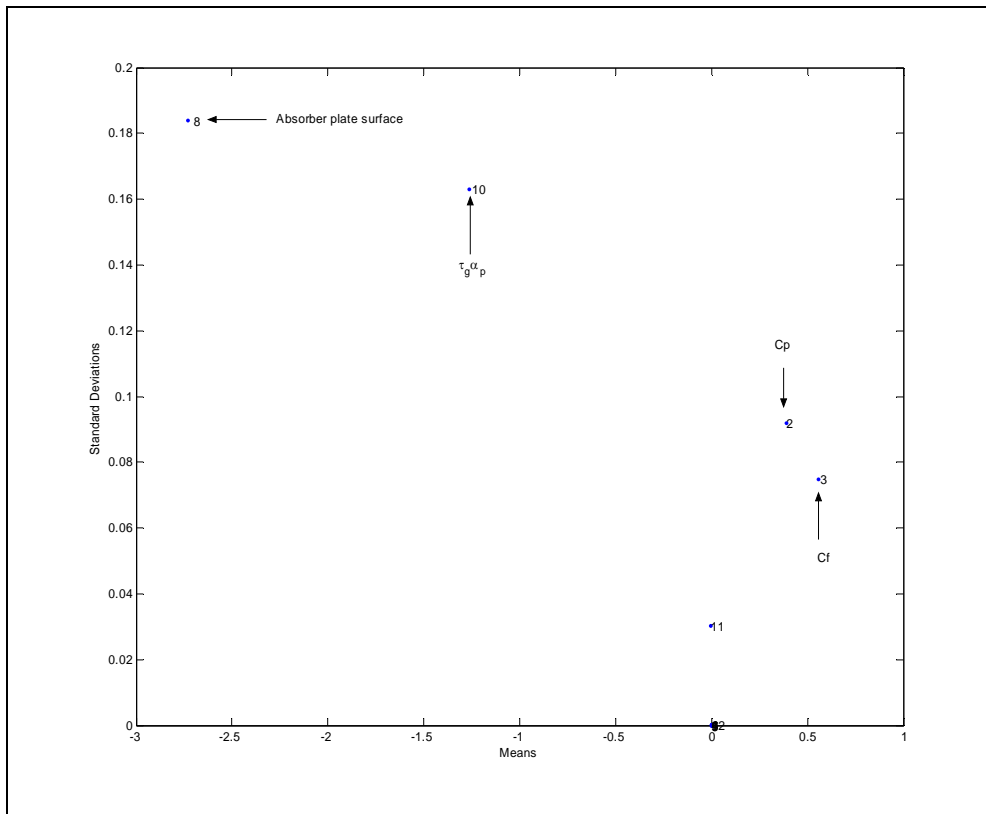
Figure 7 Distribution of elementary effects of the input factors

Factors	Range of variation	$x_{h,0}$	Frequency
C_p	[7000,9000]	8000	17
C_f	[5000,7000]	6000	29
C_g	[4500,5500]	5000	5
h_{f-p}	[100,140]	120	263
h_{g-a}	[5,15]	10	281
ε_g	[0.8, 1]	0.9	113
ε_{sky}	[0.8, 1]	0.9	131
α_g	[0,0.2]	0.05	149
S	[1,2]	1.13	181
$\tau_g\,\alpha_p$	[0.6,1]	0.88	241

Table 1 Frequency associated to the input parameters model.

Figure 8 shows the results of FAST method. Choosing odd frequency allows taking into account trigonometric properties of sinus function. The peaks generated at even frequencies are due to odd order interaction, quadratic effect or even principal effect. The odd peaks frequencies are obtained odd principal effect, or even order interaction between parameters.

The importance of a parameter is correlated with the intensity of the peak. The visual analysis of the spectrum shows clearly that the most important parameters are the transmission-absorption (241)

and the surface of the absorber plate (181). The peak at the frequency 60 is due to a second order interaction between ($\tau_g\ \alpha_p$) and S. Concerning the heat capacity of absorber and the fluid, they seem to have a little influence upon the observed output. What we observe with SA confirms the reality of the physical phenomenon. The fact that the effect of the ($\tau_g\ \alpha_p$) product has the biggest part on the global variance of output, has to be associated to the quantity of solar energy received by the absorber plate. Each parameter effect could be associated to be the physical phenomenon. In this way, it is possible to verify if all important phenomena are really taking into account. Thus, mixing the two analysis of the screening method, it is possible to classify the different factors in order of importance.

CONCLUSION

In this paper a mathematical model was developed and used for simulation of the dynamic behavior of evacuated tube collector. For the simulation, an ODE system was established and computed in MATLAB. To identify the system variables, a

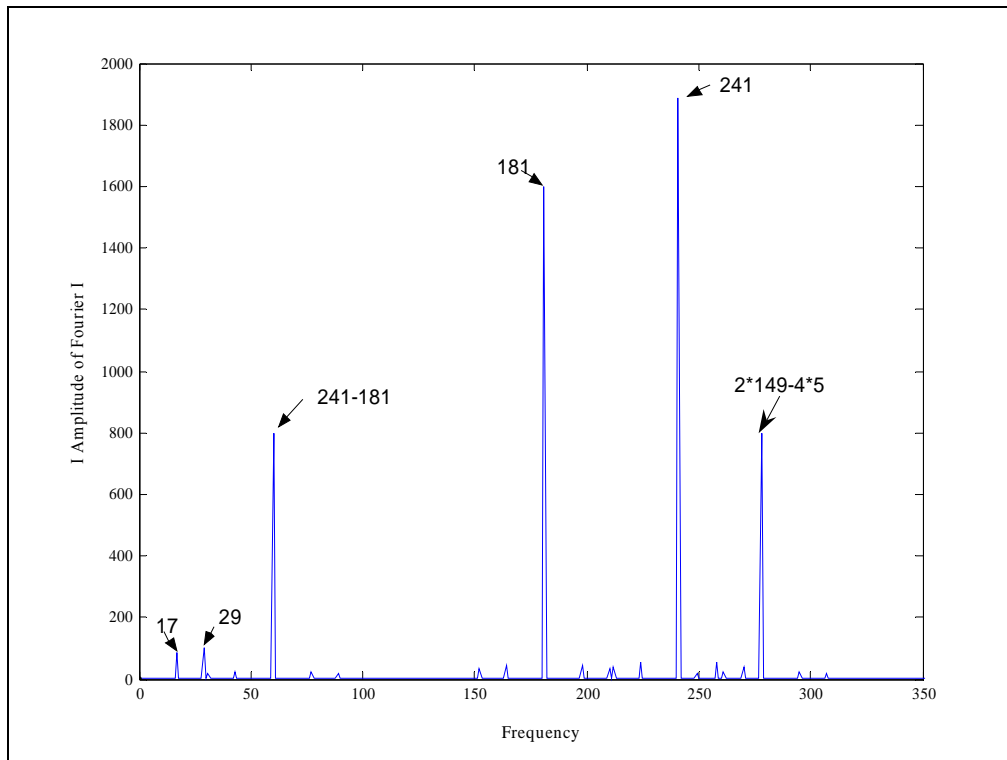measurement under natural conditions was carried out.

Figure 8 Identification according their frequency of the influential parameters

The modeling results achieved showed a fairly good coincidence with the measurements.

This measure – model prediction comparison was the first step of elements of validation.

The second approach for validation was the use of sensitivity analysis which aims to quantify the relative importance of input parameters or factors in determining their intensity. This analysis has first determined qualitatively which parameters have no impact on the output response. Then a spectral analysis established that the most important parameters is the transmission – absorption product. SA allows us to be sure that physical phenomena that are not influential in theory are not taking into account, due to the fact that all parameters can be associated to a physical phenomenon. This analysis constitutes a crucial step in providing elements of validation for the model.

ACKNOWLEGDMENTS

NOMENCLATURE

C_f fluid heat capacity (J/m².K)

C_g Heat capacity of glass cover (J/m².K)

C_p Heat capacity of absorber (J/m².K)

f Friction factor of Darcy

$G_\perp$ Global solar irradiance in the plane of the collector (W/m²)

h_{f-p} heat transfer coefficient fluid – absorber (W/m².K)

h_{g-a} heat transfer coefficient glass – ambient (W/m².K)

h_{sky} heat transfer coefficient glass – sky (W/m².k)

h_w wind convection heat transfer coefficient (W/m².k)

T_a Ambient temperature (°C)

T_f Fluid temperature (°C)

T_g Temperature of glass cover (°C)

T_p Absorber temperature (°C)

T_{sky} Sky temperature (°C)

u fluid velocity (m/s)

v wind velocity (m/s)

Greek symbols

α   absorptivity coefficient

Δ   variation on input parameters

ε   emissivity

σ   Stefan-Boltzmann constant (5.67×10^{-8} $W.m^{-2}.K^{-4}$)

σ_i Standard deviation

Φ_{LW} Long wave flux (W/m²)

ρ_d Diffuse reflectance of the cover system

μ_i means of parameters values

Subscripts

f   fluid

g   glass

p   absorber plate

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