Characterisation of Remote Loading Systems using a Nonlinear Analytical and Numerical Method

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Abstract – Remote loading systems using an electromagnetic beam is affected by discontinuous piecewise constant loading voltage. The characteristics of these remote dynamic systems under charge are investigated. Varied behaviour of the systems with discontinuous charge is found due to environment perturbation compared to systems under continuous voltage charge with a wired link. A theoretical and numerical analysis is done in order to obtain the responses of the systems to various piecewise constant charge voltage under different initial conditions. Also, for such piecewise systems, the conditions of oscillation and the stability are given.

Keywords – Remote energy saving, nonlinear-dynamic loading systems, discontinuous charge voltage, piecewise constant variables, oscillation, numerical simulation.

I. INTRODUCTION

Generally, in many remote electronic systems, loadings acting on the remote systems are discontinuous and can be considered as piecewise constants in first approximation. But, the energy stocking could be often chaotic or non linear and is produced by piecewise discontinuous disturbances. Due to the characteristics of the discontinuous charge, loading of a dynamic system disturbed by the piecewise constant voltage shows an entirely different behavior from that of the corresponding continuous system. For example, the loading of the systems acted by piecewise constant loaded voltage can be very sensitive to initial conditions even for the linear dynamic systems. The peculiar behavior will be demonstrated for several linear and nonlinear dynamic systems under piecewise constant charges.

In a wireless sensor network, the dynamic system will be an intelligent node with an autonomous battery. This last is a remote loading with the help of an electromagnetic beam. Several mathematical methods exist to characterize these phenomena. Differential equations with piecewise constant arguments of retarded and advanced types such as $q([t])$ or $q([t\pm n])$ were proposed [1-8]. However, in the current literature, there is still a lack of systematic studies on the properties of charging of the remote dynamic systems subjected to piecewise loading voltage. A novel piecewise constant argument $[Nt]/N$ for analytically and numerically solving the second order differential equations which govern the nonlinear dynamic systems exerted by piecewise constant voltages is introduced [8]. With this piecewise constant argument, the gap between the dynamic systems subjected to continuous loadings and the systems under piecewise charges was filled.

With the help of the piecewise constant argument $[Nt]/N$, the present work investigates the dynamic systems subjected to piecewise constant loading voltage with focus on the extraordinary behaviour of remote charging systems. Theoretical analysis of the properties of loading of remote dynamic systems under a piecewise constant charge voltage will be undertaken. For the safe of clarification the results of the corresponding loadings of the systems will be studied numerically and graphically with various combinations of coefficients in the equations of loading and different initial conditions so that the behaviour of the systems under piecewise constant voltage may be visualized and comprehensively understood. Throughout the present work, the exponential matrices to be derived characterize the oscillatory behavior of a system subjected to piecewise constant voltage.

This paper is organizer as follow. Mathematical formulation of the remote loading systems is expressed in section II for an approximate solution to a strongly non-linear second order differential equation [9-12]. Then, in section III, the loading of remote dynamical systems under sinusoidal piecewise constant voltage is studied and shown a divergent oscillatory behaviour. A stable remote loading system with constant amplitude could be obtained if the remote system gets low capacitance (section IV). In the section V, oscillatory loading of the dynamic systems will then be examined with the help of the diagrams of charge and piecewise constant voltage charge against time. The solutions obtained for the systems will be compared with those of the corresponding continuous systems. Finally, conclusions are made.

II. MATHEMATICAL FORMULATION

Characterization of a remote loading system could be physically governed by a differential equation (1) with piecewise constant arguments (PCA) originated by Cooke, Shah and Wiener [2, 7] reformulated in our case as

$$L\ddot{q} + R\dot{q} + C^{-1}q = F(t) \quad (1)$$

where

- L is the inductance of the system,
- R is the resistor coefficient,
- C is the capacitance,
- $F(t)$ is an excitation function.

Here, the relation $R^2 > 4LC^{-1}$ is assume in equation (1). In our case the PCA becomes an equation with piecewise constant voltage (PCV). The value of the PCV shown in the equation (1) can be calculated for any given time. The analysis of the properties of loading starts with these initial conditions

$$q(t=t_0) = q_0 \text{ and } \dot{q}(t=t_0) = i(t=t_0) = i_0 \quad (2)$$

The paper first received April 2008 and in revised form June 2008.
Digital ref: Al70301046

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Let's choose an arbitrary time segment $[Nt]/N \le t < ([Nt]+1)/N$ where $[\bullet]$ represents a function of greatest integer and N is an integer which controls the size of the time period on which the external charge is a constant. Then, solution of the linear system described by equation (1) is readily available. This entails that the loading in any time interval represents a portion of a linear charging. At the ending point of an interval, the charge and current of the system are given by $q(([Nt]+1)/N)$ and $\dot{q}(([Nt]+1)/N)$ respectively.

For the loading of the next time interval, these particular results become in turn the starting conditions or the local initial conditions. The loading in the follow-up intervals can then be consequently obtained with the local initial conditions and so on.

Considering that the remote loading of the system is continuous, i.e., $q(t)$ and $\dot{q}(t)$ are continuous on $t \in [0,\infty]$, the following conditions of continuity must be satisfied

$$\begin{cases} q_{[Nt]}\left(\dfrac{[Nt]}{N}\right) = q_{[Nt]-1}\left(\dfrac{[Nt]}{N}\right) \\ \dot{q}_{[Nt]}\left(\dfrac{[Nt]}{N}\right) = \dot{q}_{[Nt]-1}\left(\dfrac{[Nt]}{N}\right) \end{cases} \tag{3}$$

III. Loading of Remote Dynamical Systems Under Sinusoidal PCV

Considering, the remote loading system is excited by a piecewise sinusoidal voltage, the following function must satisfied $F(t) = \rho\cos\left(\omega\dfrac{[Nt]}{N}\right)$ where ρ is the amplitude of the piecewise constant charge. So, the remote system is acting by a sinusoidal varying PCV. It is independent of the charge $q(t)$. The solution of equation (1) for the entire time range considered can be derived as equation (4) with the continuity conditions and the solutions for each of the time intervals

$$q(t) = e^{-\alpha\left(t-\frac{[Nt]}{N}\right)} \cdot A \cdot B + \frac{\rho}{\omega_0^2}\cos\left(\omega\frac{[Nt]}{N}\right)e^{-\alpha\left(t-\frac{[Nt]}{N}\right)} \cdot \left[e^{-\alpha\left(t-\frac{[Nt]}{N}\right)} - \cos\left(\tau\left(t-\frac{[Nt]}{N}\right)\right) - \frac{\alpha}{\tau}\sin\left(\tau\left(t-\frac{[Nt]}{N}\right)\right)\right] \tag{4}$$

where $\omega_0 = (LC)^{-\frac{1}{2}}$, $\alpha = \dfrac{R}{2L}$, $\tau = \sqrt{\omega_0^2 - \alpha^2}$ and the matrix A is written as

$$A^T = \begin{bmatrix} \cos\left(\tau\left(t-\frac{[Nt]}{N}\right)\right) + \frac{\alpha}{\tau}\sin\left(\tau\left(t-\frac{[Nt]}{N}\right)\right) \\ \frac{1}{\tau}\sin\left(\tau\left(t-\frac{[Nt]}{N}\right)\right) \end{bmatrix} \tag{5}$$

Also, the matrix B in equation (4) takes the following form

$$B = e^{-\alpha\frac{[Nt]}{N}} D^{[Nt]}\begin{bmatrix} q_0 \\ i_0 \end{bmatrix} + \sum_{m=1}^{[N_t]} \frac{\rho}{\omega_0^2} e^{-\alpha\frac{m}{N}} D^{m-1}\begin{bmatrix} e^{\frac{\alpha}{N}} - \cos\frac{\tau}{N} - \frac{\alpha}{\tau}\sin\frac{\tau}{N} \\ \left(\frac{\alpha^2}{\tau}+\tau\right)\sin\frac{\tau}{N} \end{bmatrix}\cos(\omega([Nt]-m)) \tag{6}$$

And the square matrix D has the form like

$$D = \begin{bmatrix} \cos\frac{\tau}{N} + \frac{\alpha}{\tau}\sin\frac{\tau}{N} & \frac{1}{\tau}\sin\frac{\tau}{N} \\ -\left(\frac{\alpha^2}{\tau}+\tau\right)\sin\frac{\tau}{N} & \cos\frac{\tau}{N} - \frac{\alpha}{\tau}\sin\frac{\tau}{N} \end{bmatrix} \tag{7}$$

A simple case of remote loading without damping is studied by substituting the values of system parameters and time into the analytical solution equation (4). Then, the remote loading of the systems governed by equation (1) could be seen in figure 1 with the help of [13].

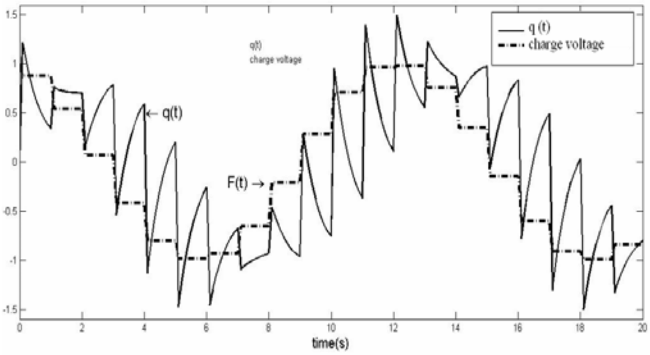

Fig. 1 : Charge and piecewise constant voltage acting on a remote loading system versus time with L=1, R=1, N=1, C=1, $\rho = 1$, $\omega = 1$

Fig. 1 expresses the charge versus time and also the piecewise constant voltage based on the solution of equation(1). One seen that the curve of charge is continuous everywhere for $t \ge 0$ under the discontinuous piecewise constant charge. Moreover, the charge shows slope discontinuities at the integer points of time. The discontinuities are a consequence of the discontinuous piecewise constant voltage acting on the system. In addition, charge plot follows asymptotically an oscillatory behavior. When *F(t)* decreases from maximum peak to the minimum peak, the remote system tends to take charge. Unlike when *F(t)* increases, the system tends to unload. It could be seen one second delay between the charge and *F(t)* due to the inertia of the system to load.

IV. Stable Remote Loading System with Constant Amplitude

Under certain conditions, the remote dynamic system subjected to piecewise constant voltage may behave as a simple loading system with stable remote loading of constant amplitude. Here a similar system is considered as section III, but with a higher C ($C > 1$).

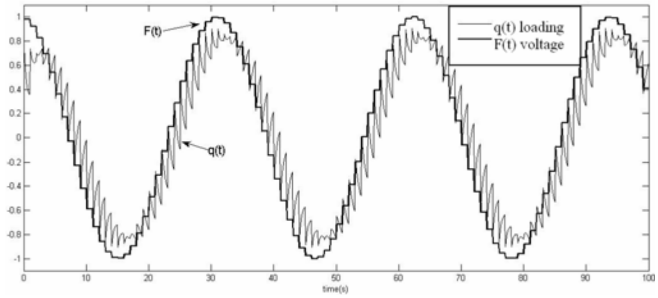

Fig. 2 : Charge and the corresponding piecewise constant voltage acting on a remote loading system with L=4, R=3, N=1, C=5, $\rho = 1$, $\omega = 0.2$.

From figure 2, this remote loading system shows a stable oscillatory loading under the same conditions as previous section and after a relatively long time from t=0. Even though the shapes of the stepwise charges are different from period to period, the waveforms of the charge are repeating precisely with a period of 2π. When the external piecewise constant charge is charge related, the complete solution for a time interval of the system must first be obtained so that the end conditions for this interval, and the starting conditions of the consecutive interval, may be determined accordingly. Also, a higher C makes stable the remote loading system with constant amplitude.

V. Remote Damped Loading System Subjected to a PCV

Generally, loading of a remote dynamic system subjected to a PCV displays a dramatically different behavior from those of the corresponding system exerted by continuous voltage. Compare a damped loading system subjected to a PCV described by the equation of charge:

$$L\ddot{q}+R\dot{q}+C^{-1}q=\rho q\left(\frac{[Nt]}{N}\right) \tag{8}$$

with a similar linear loading system governed by the following equation of charge

$$L\ddot{q}+R\dot{q}+C^{-1}q=\rho q(t) \tag{9}$$

where R, L, C and ρ are constants of the system's physical properties. Equations (5) and (6) verify $R^2>4LC^{-1}$. The solution of the linear system in equation (6) is readily available and the solution of equation (5) can be derived as follows:

$$q(t)=e^{-\alpha t}\left\{(1-\rho C)\left[\cos\left(\tau\left(t-\frac{[Nt]}{N}\right)\right)+\frac{\alpha}{\tau}\sin\left(\tau\left(t-\frac{[Nt]}{N}\right)\right)\right]\right.$$
$$\left.+\frac{\rho C}{\tau}e^{-\alpha\left(t-\frac{[Nt]}{N}\right)}\sin\left(\tau\left(t-\frac{[Nt]}{N}\right)\right)\right\}G^{[N_t]}\begin{bmatrix}q_0\\ i_0\end{bmatrix} \tag{10}$$

In equation (10), the constants α and τ are the same as those defined in equation (4) and the square matrix G takes the form

$$G=\begin{bmatrix}\rho Ce^{-\alpha}+(1-\rho C)\cos\left(\tau-\frac{\alpha}{\tau}\right)\sin\tau & \frac{\sin\tau}{\tau}\\ -(1-\rho C)\left(\frac{\alpha^2}{\tau}+\tau\right)\sin\tau & \cos\left(\tau-\frac{\alpha}{\tau}\right)\sin\tau\end{bmatrix} \tag{11}$$

A. Simple case without resistive load

A no resistive loading remote system is governed by the following differential equation

$$L\ddot{q}+C^{-1}q=\rho q\left(\frac{[Nt]}{N}\right) \tag{12}$$

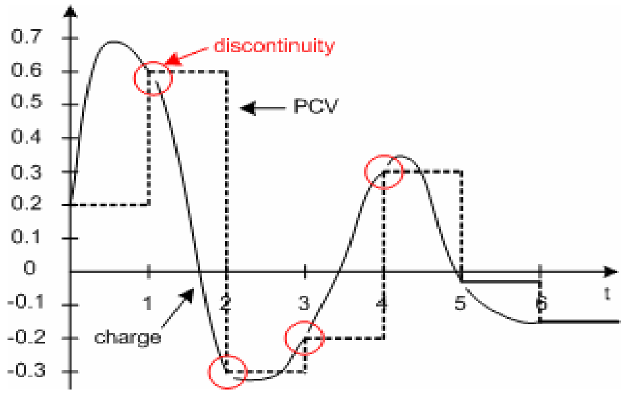

Fig. 3 : Charge and PCV acting on a remote damped loading system versus time with L=1, R=0, N=1, C=0.25, $\rho=1$, $\omega=0.2$

The charge of such remote loading system shows discontinuity at the integer point of time (Fig. 3). As displayed, the PCV exerting on the system has the same value as the charge at the integer point of time, and the corresponding loading in this case is asymptotically convergent.

B. Comparison of constant amplitude continuous loading system versus PCV loading system

Continuous system and the PCV remote loading system is compared under constant amplitude (Fig. 4). The loading with the PCV $F(t)=-10q\left(\frac{[Nt]}{N}\right)$ diverges asymptotically. Conversely, the charge under the continuous voltage $-10q(t)$ is damped out rapidly with time.

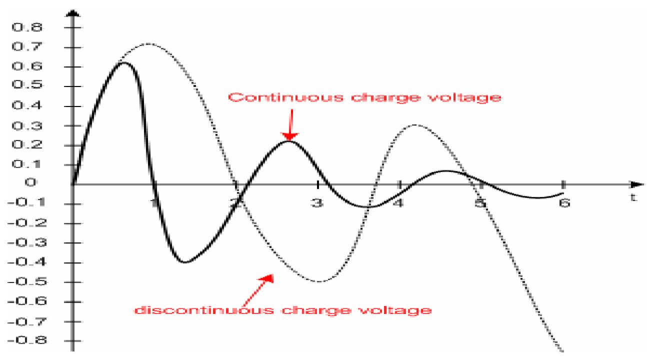

Fig. 4 : Comparison of the loading of the system subjected to PCV versus continuous charge voltage with L=1, R=1, N=1, C=0.66, $\rho=-10$

C. Comparison of sinusoidal continuous loading system versus sinusoidal PCV loading system

For the remote loading systems subjected to known forms of piecewise constant and continuous voltage respectively, such as $\rho\cos\left(\omega\frac{[Nt]}{N}\right)$ and $\rho\cos(\omega t)$ which are merely time dependent, the comparison is also evident. As an example, the lading of the system governed by equation (1) is compared with the following system subjected to a continuous voltage,

$$L\ddot{q}(t)+R\dot{q}(t)+C^{-1}q(t)=\rho\cos(\omega t)\,, \tag{13}$$

as shown in Fig. 5. It can be observed that the loading of the system subjected to the continuous voltage and system under the PCV oscillates and becomes a damped steady-state oscillation with identical shape as time increases up to t=4s. After this time, the waveform of the PCV loading system becomes divergent.

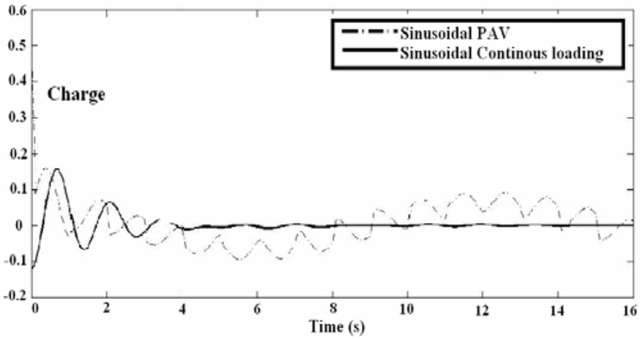

Fig. 5 : Comparison of the loading of the system subjected to piecewise constant voltage versus continuous charge voltage with L=1, R=0.5, N=1, *C*=20, $\rho = 1$, $\omega = 0.5$

VI. Conclusion

This study is conducted to investigate the characteristics of the remote loading of the linear and nonlinear dynamic systems subjected to piecewise constant charges voltages. Solutions for the remote dynamic systems with PCV are developed with the piecewise constant argument $\frac{[Nt]}{N}$. Moreover, the systems expressed in equations (1) and (5) may also be continuous systems when N tends to infinity as described in [3]. The solutions in the form of equations (4) and (10) with finite N values are unique and become the specific piecewise constant systems as those studied in [1, 2, 5]. As such, the solutions in the forms of equations (4) and (10) bridge the gap between the piecewise constant systems and continuous systems. Extraordinary behaviours of the remote dynamic systems are found in the study in comparing with the dynamic systems under continuous voltages, hence the difficulty in finding appropriate physical interpretation of the model.

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Biographies

N. M. Murad was born in Saint Denis of La Réunion, France in 1974. He received his PhD in communication & electronics at the Ecole Nationale Supérieure des Télécommunications (Telecom Paris), in 2001. From 1998 to 2001, as an engineer R&D in radio operator mobile it worked at Alcatel CIT where it prepared his doctoral thesis with Télécom Paris in parallel.
Between 2001 and 2003, he worked as a teacher and research at the University of La Reunion where he had co-responsibility on the coupling between energy and telecommunication project with Dr. Celeste. During 2004 to 2007, he is an assistant professor researcher at the 3IL school of engineer and researcher at the XLIM/OSA.
Since September 2007, he is an assistant professor researcher at the University of Reunion. Its research relates to the numerical communication, the signal and information theory with a specific accent on the wireless communications (UMTS, CDMAone, LMDS, 802.16,…), spread spectral techniques, synchronisation, MIMO system, spatial and polarization diversity, wireless energy transportation, wireless mobile network and wireless sensor network.

Alain Celeste was born in Lyon, France, in 1965. In 1987, he graduated as an Engineer in Physics from the National Institute of Applied Sciences (INSA), in Toulouse, France. In 1990, he received a Ph.D. in Solid State Physics from the University of Toulouse, for his work on resonant tunnelling in semiconductor heterojunctions. He then started to work on microwave characterization of dielectric materials using waveguide loading techniques, during his military service. Since 1994, he is an Assistant Professor at the University of La Reunion, in charge of the Wireless Power Transmission research field and teaching digital electronics and microprocessor architecture. His current interests are wireless power transmission for remote power provision to smart sensors and communicating devices.